Impact of Partial Update on Denoising Algorithms of ECG Signals

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***Abstract*—This work aims to propose and study the effects of partial update procedure on various ECG denoising algorithms. The partial update algorithms are applied to overcome different types of noises such as Power-Line Interference (PLI), Baseline Wander (BW), Electrode Motion artifacts (EM) and Muscle Artifacts (MA). The impact of partial update (PU) on multiple algorithms and spatially adaptive filters and multi-layer Neural Network (NN) are studied and demonstrated. The performance of different algorithms are evaluated by measuring the Signal-to-Noise Ratio after cancellation (Post-SNR), the Mean Square Error (MSE) and the Percent Root Mean Square Difference (PRD%).**

***Index Terms*—Partial Update; ECG Noise Canceller; Neural Network; Adaptive Filters; Mean Square Error.**

I. INTRODUCTION

Heart signal is an important and essential indicator for doctors to diagnose diseases in this important human body organ. This signal is an electrical signal that clarifies the activity of the heart muscles, and so it is important for a doctor to obtain this signal free of noise which hinders him to diagnose the diseases accurately. We infer information of heart-related diseases from amplitude and duration of ECG waves from P to U wave. ECG signals are measured by the electrodes placed on the human body, and often corrupted by various artifacts that change the original signal. Therefore, we need to remove these artifacts from ECG signals. There are many types of noise that corrupt ECG signals, but the common artifacts present in the ECG signals include Power-Line Interference, PLI, related to the noises come from power, Baseline Wander, BW, Electrode Motion artifacts, EM and Muscle Artifacts, MA. These three artifacts or noises are related to the process of acquisition [1].

The frequency band of PLI is (0.05-100 Hz) near to the frequency band of ECG signals; this is the main source of interference. Drift of the baseline during respiration is the source of BW. It is considered as a non-stationary sinusoidal signal with the respiration frequency and amplitude varied in time [2]. The most common noises in ECG signals measurements are EM and MA. EM artifacts are induced by the electrode-skin impedance if electrode motion happens. MA artifacts are induced because of the contraction of skeletal muscles which appear due to the patient's movement [3]. Due to the randomness of noises in nature and wide range of frequency band located in the frequency band of the heart signal, filtering these artifacts from the ECG signals becomes a challenge, and it is considered as an essential purpose for a diagnosis process.

During past few years, various algorithms and techniques have been used for denoising ECG signal [4-17]. Adaptive filters are one of the methods used for this purpose. Adaptive Noise Cancellation, ANC, is a technique and an algorithm for estimating the input signals and extracting from noises effectively. The feature is that levels of noise cancellation are achievable that would be difficult or impossible to achieve by other signal processing means of canceling noise, without the need to know the signal statistical characteristic or noise. One of the common techniques is the least mean square, LMS, algorithm. It is used to minimize errors between target signal and the output performance of the linear filter by recursively adjusting the linear filter parameters.

An improvement for LMS is Recursive least mean square, RLS, algorithm; it is an improvement of a computational complexity RLS filter. It covers the convergence of magnitude to be faster than that of LMS filter, based on the inverse correlation matrix of signal data.

An important filtration adaptive algorithm and one of the famous techniques is Artificial Neural Network, ANN, or generally called neural network, NN. It is a computational model that combines an interconnected group of artificial neurons. A conventional feed forward multi-layer NN is usually driven by the well-known BP algorithm.

This work aims to propose a new augmentation technique; it implements and studies the Partial Update, PU, technique impact after applying or augmenting it to previous algorithms to decrease the complexity of the filtering process and enhance the performance of the denoising process. Signal-to-Noise Ratio after cancellation (Post-SNR), the Mean Square Error (MSE) and the Percent Root Mean Square Difference (PRD %) are considered and used for the performance evaluation of the proposed algorithms and the impact of partial update technique. In the following section, algorithms and augmentation process are introduced.

II. METHODS AND AUGMENTATION PROCESS

The sequence of the proposed work is implemented based on the augmentation of PU with LMS, RLS and NN. The flow diagram is illustrated in figure 1. ECG records taken from MIT-BIH arrhythmia database are used [21] where 3600 samples of the ECG signals are operated. The ECG signal frequencies are between 0.5 Hz and 100 Hz [22]. The main target is the augmentation of PU with NN and comparison with LMS and RLS in the same experiments. A real BW, MA and EM noises are used and loaded from MIT-BIH; it is Noise Stress Test Database, NSTDB. In the case of PLI, PU is used for decreasing the complexity.

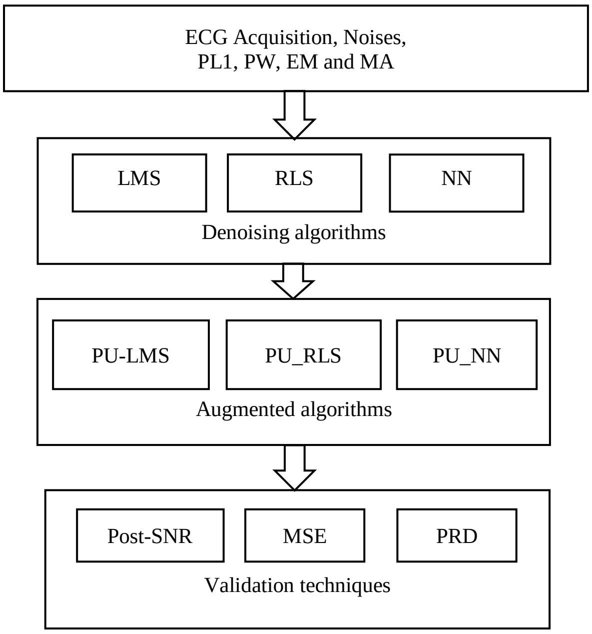

Figure 1: Flow diagram of the proposed algorithm

A. Partial Updates

In general, more hardware multipliers, adders and memories imply more power consumptions. The main reason for using PU is to limit hardware multipliers that cause high power consumption. PU techniques keep up the convergence rate of the common algorithms and, sometimes, better than the same algorithm that doesn't use PUs. The number of PU coefficients should be considered when doing PUs. There are many types of Pus, such as periodic PUs, sequential PUs, Stochastic PUs, M max updates, selective PUs, set membership PUs and block PUs [20]. The PU technique has the weights updating process be on and off, meaning that we update the weights for some samples and stop updating some other samples in a periodic manner. Periodic PU is used for all used algorithms either adaptive filters or NNs because the ECG signals are quasi-periodic, i.e., ECG signals wave shape is recurrent almost periodically. The method of periodic PUs allows the update complexity to be spread over a number of samples to reduce the average update complexity per sample.

In the following Section B, the most common adaptive filters, LMS and RLS, are introduced, and the impact of PU for these techniques is implemented. In Section C, NN is introduced. The main target for this work is to propose the augmentation technique, PU, for LMS, RLS and NN, to study and implement the effect of all improvement for all noises and to compare the improvements with respect to validators. These validators are described in Section D.

B. Adaptive learning Denoising algorithms

The weight vectors of adaptive algorithms are updated to minimize and optimize the cost function. LMS and the RLS algorithms are considered as linear adaptive filter algorithms. Different modifications for both algorithms have been manipulated in previous studies. These techniques are introduced and augmented by PU to infer their performances in the filtration process for different noises. Next, all results are compared with proposed algorithms of PU with NN.

i. LMS and PU_LMS

The LMS algorithm is considered as an adaptive algorithm based stochastic gradient algorithms. It changes the filter tap weights so that $e(n)$ is minimized in the mean-square sense. PU is applied to LMS, and the weight-update function of a typical adaptive filter can be written as:

$$w(n+1) = w(n) + \Delta w(n) \tag{1}$$

The PU method chooses M elements from the difference weight $\Delta w(n)$ and generates new weights. It modifies Equation (1) to:

$$w_i(n+1) = \begin{cases} w_i(n) + \Delta w_i(n) & if\ i \in I_M(n) \\ w_i(n) & otherwise \end{cases} \tag{2}$$

where w_i means the i^{th} element of w and $I_M(n)$ is a subset of $\{1, 2, \ldots, \mathrm{N}\}$ with M elements at time n. For different PU methods, the subset $I_M(n)$ is different at different time [11-12].

Since a specific PU method in one adaptive filter algorithm which achieves good performance may not perform well in another adaptive filter algorithm, the performance of one PU method for different adaptive filter algorithms is also compared. The periodic PU method only updates the coefficients at every S^{th} sample and copies the coefficients at the other samples, where $S = \left\lceil \frac{N}{M} \right\rceil$ which is the ceiling of $\frac{N}{M}$. where N is the number of samples. The update function can be written as:

$$w(S(n+1)) = w(Sn) + \Delta w(Sn) \tag{3}$$

This method can reduce the overall computational cost. Since periodic PU algorithms update the whole vector, the steady-state performance is the same as the original adaptive filter algorithms for stationary input. Periodic PU algorithms have convergence S times slower than the basic or original algorithms. The weight-update function of PU-LMS is:

$$w_i(n+1) = \begin{cases} w_i(n) + \mu\, e_i(n) x_i(n) & if\ i \in I_M(n) \\ w_i(n) & otherwise \end{cases} \tag{4}$$

ii. RLS and PU_RLS

Unlike LMS, RLS input signals are considered as deterministic signals, compared to most common competitors, the RLS exhibits fast convergence. The rate of convergence is invariant to the eigen value spread of the correlation matrix of the input vector [13]. It works in time-varying environments with the cost of an increased computational complexity and some stability problems. The cost of function, C, for RLS is defined as:

$$C = \sum_{i=0}^{n} \lambda^{n-i} e^2(i) \tag{5}$$

where λ is defined as the forgetting factor which gives exponential weights to older error samples. The cost function C is dependent on coefficients $w(n)$. The cost function C is minimized by taking partial derivative with respect to the filter coefficients $w(n)$. The weight-update function of PU-RLS is:

$$w_i(n) = \begin{cases} w_i(n-1) + e_i(n) g_i(n) & if\ i \in I_M(n) \\ w_i(n-1) & otherwise \end{cases} \tag{6}$$

C. NN and PU_NN

In this method, the case is different where the weights of this proposed NN are adjusted at the same time the input signal is being processed (continuous learning) as shown in Figure 1. The modification in step size is given by:

$$\mu_{new} = \mu_{old}\left(\frac{1}{ai^2 n+1}\right) \quad (7)$$

The equation of weights between input and hidden layers for VS-NN is updated as follows:

$$w_{kj}(n+1) = w_{kj}(n) + \Delta w_{kj}(n) \quad (8)$$

where the change in the weights is given by:

$$\Delta w_{kj}(n) = \mu_{new}\delta_k(n)x_j(n) \quad (9)$$

The equation of weights between hidden and output layers for VS-NN is updated as follows:

$$w_{ok}(n+1) = w_{ok}(n) + \Delta w_{ok}(n) \quad (10)$$

where the change in the weights is given by:

$$\Delta w_{ok}(n) = \mu_{new}\delta_o(n)x_k(n) \quad (11)$$

where the weights w_{kj} and w_{ok} are the synaptic weights between the input j and hidden k layers and the synaptic weights between the hidden k and output o layers of NN respectively. And where $\delta_k(n)$ and $\delta_o(n)$ are the local gradient of the input layer and of the hidden layer respectively, a is parameter, n is the sample number and i is the iteration number.

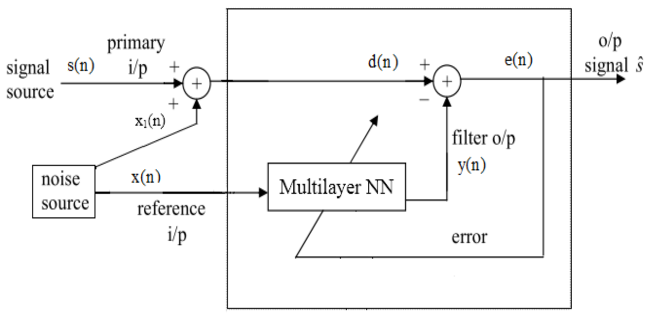

Figure 2: The modified NN in continuous learning

Appling the weight-update function of PU-NN is:

$$w_{kji}(n+1) = \begin{cases} w_{kji}(n) + \mu\left(\frac{1}{a i_n^2 n + 1}\right)\delta_k(n)x_{ji}(n) & if\ i \in I_M(n) \\ w_{kji}(n) & otherwise \end{cases} \quad (12)$$

where $\delta_k(n)$ is the local gradient of hidden layer, a is constant, in is the iteration number, n is the sample number and μ is step size (learning rate).

D. Validators metrics

Post-SNR dB, MSE and PRD% are calculated and considered as validators, and metric for evaluation process of the proposed algorithms and the computation comparisons are illustrated as follows [17].

$$Post-SNRdB = 10\log\frac{\sum_{n=1}^{N}s^2(n)}{\sum_{n=1}^{N}[e(n)-s(n)]^2} \quad (13)$$

$$MSE = \frac{1}{N}\sum_{n=1}^{N}[e(n)-s(n)]^2 \quad (14)$$

$$PRD\% = \sqrt{\frac{\sum_{n=1}^{N}[e(n)-s(n)]^2}{\sum_{n=1}^{N}s^2(n)}}\times 100 \quad (15)$$

where the ANC system has primary and reference inputs. In the beginning, the input receives a signal $s(n)$ from ECG signals source added with noise $x_1(n)$ not correlated with the signal source. The noise $x(n)$ moves through a filter to produce an output $y(n)$ that is approximately equal to primary input noise. This noise estimate $y(n)$ is subtracted from the corrupted signal $d(n)$ to induce an estimate of the signal $e(n)$, the ANC system output is supposed to be the same and similar to the clean main signal $s(n)$ [22].

III. Impact of PU on Performance of Noise Cancelation Algorithms Based on Simulation Results

Through simulation experiments, iterations are taken to be 150 iterations, 540000 samples, or 300 iterations, 1080000 samples, when 22 ECG records taken from MIT-BIH arrhythmia database are used. The Imp-SNR dB of MVSS is negative in the ECG records of 112 and 118

A. Impact on PLI cancellation

Frequency band 0.5Hz-100Hz of ECG signals are very low. This low frequency interfered by PLI of 50Hz noise. This noise is also the source of interference for ECG signal recording. So this 50Hz noise corrupts the output of ECG signal. Notch filter is used to remove the noise at 50Hz. However, the power supplies of hospitals have slight variations. The practical frequency of the power supply hypothetically might vary between 47Hz and 53Hz. A static or normal filter has to remove all frequencies including signals frequencies between 47Hz and 53Hz, and this led to degrade and decrease the efficiency of the ECG recordings excessively [14].

For the ECG simulations, a noise-free has been generated on ECG signal upon ECG function in MATLAB and then mixed with power line signal 50Hz. The ECG signals as shown and described in Figure 3.

The characteristics of ECG signal is that very weak time varying signal and has a frequency between 0.5 Hz to 100 Hz. The frequency spectrum of unfiltered ECG signals plot is displayed in Figure 4, and it has a spike at a frequency of 50 Hz, which indicates that there is interference at ECG frequency bands.

The specified simulation of LMS algorithm is performed with order M=18, step size μ = 0.015, 0.008, 0.005 and 0.014, and the iterations N=1000 for the proposed filter. The output results for LMS are in the following Figure 5.

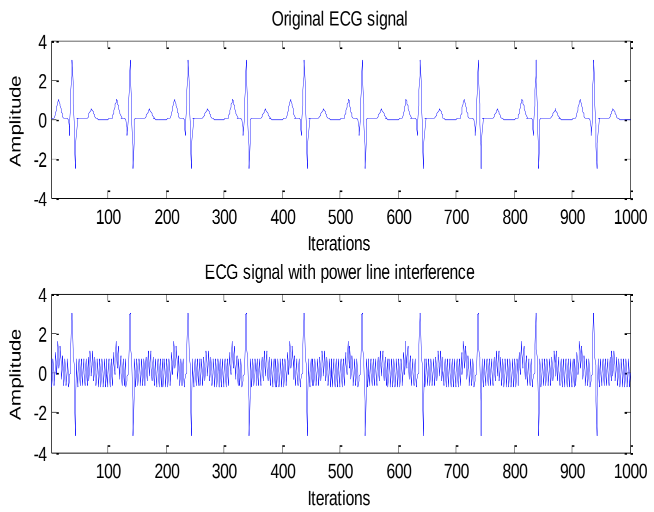

Figure 3: Complete noise free ECG signal and ECG signal corrupted with PLI

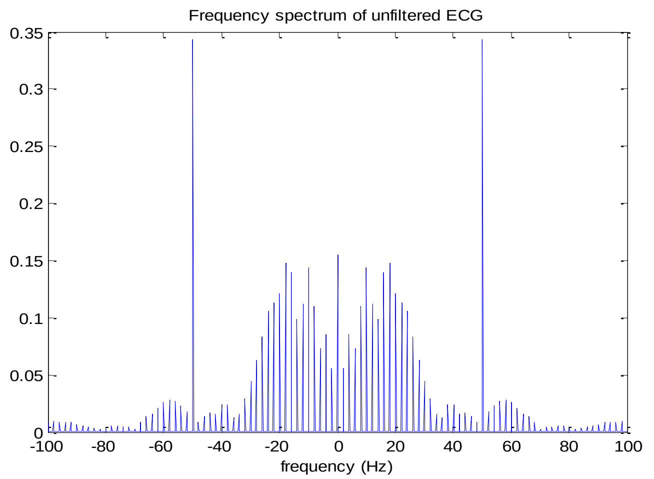

Figure 4: Frequency spectrum of unfiltered ECG

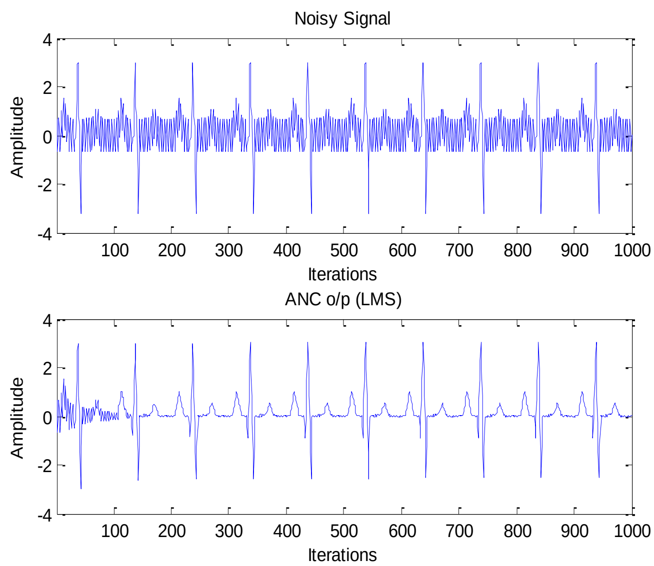

Figure 5: MATLAB simulation for LMS algorithm

Figure 6 shows the different signals between the original and restored signals due to various algorithms. It shows that SNR dB of the PU_NN is better than the SNR dB of LMS algorithm, and both are higher than SNR of RLS, RLS has no relevant values as shown in the following graph.

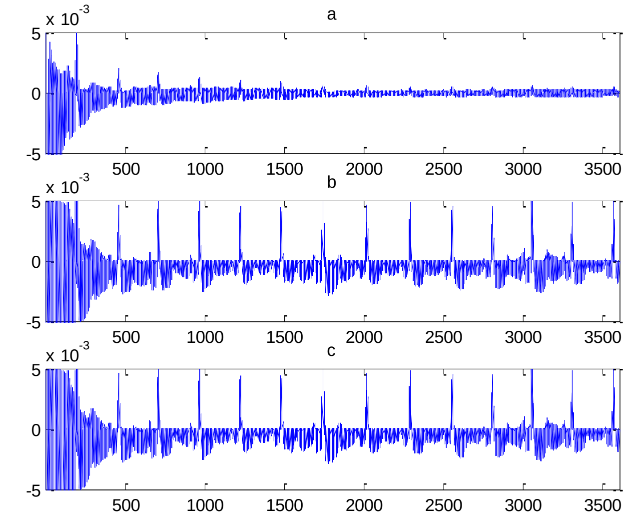

Figure 6. Typical filtering results of PLI cancellation, amplitude versus samples (a) difference signal after RLS filtering, (b) difference signal after MSE filtering, (c) difference signal after PU-NN filtering

The outputs of filtered signals after noise cancellation in frequency domain is shown in Figure 7.

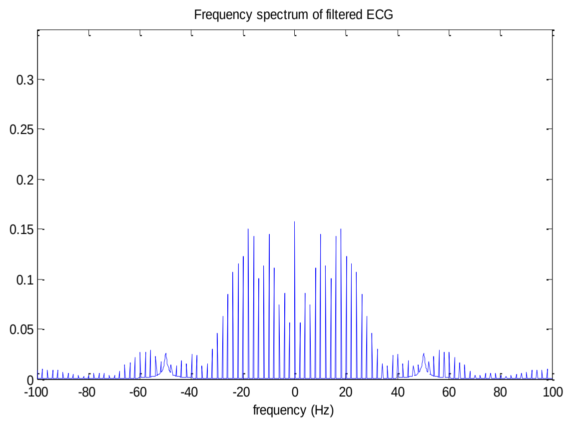

Figure 7: Frequency spectrum of filtered ECG signal

Based on the three validators or metrics, values are shown in Table 1. RLS has no relevant values in PLI, while values for both POST-SNR dB, MSE and PRD are better for PU_NN than ones related to LMS.

Table 1
Performance contrast of various algorithms for PLI removal

From ECG Database		Validators	
Algorithm	Post-SNR dB	MSE	PRD%
PU-NN	110.119	2.28×10-6	0.4063
PU-LMS	37.037	0.0034	15.695

B. BW reduction

For the cancellation of BW noise, 3600 samples of the ECG signal that we obtained from MIT-BIH Arrhythmia Database corrupted with real BW are used. In Figure 8, the input to the adaptive filter is the corrupted ECG signal; it is considered as the primary input.

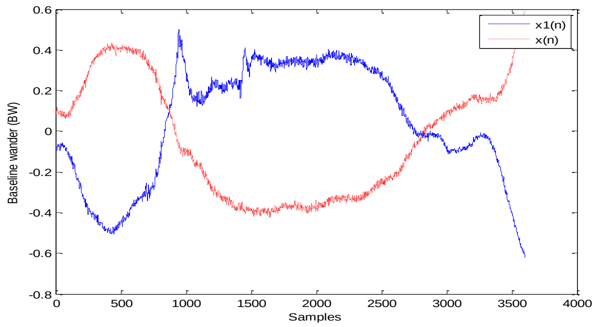

Figure 8: The primary and reference noises for BW

In the BW noise case, we evaluate the different algorithms under two pre-SNR dB: one low Pre-SNR =-5.42 dB and one high Pre-SNR=5 dB where represent the input signal to noise ratio. BW has been taken as the reference input to adaptive filter $x(n)$ where $x_1(n)$ represents the primary input to adaptive filter as shown in Figure 8.

Table 2 shows the results of the performance contrast of various algorithms for removing BW from ECG signal at 300 iterations. PU-LMS algorithms have low MSE in comparison to other algorithms, whether in low or high pre-SNR dBs. Partial updates for NN, PU_NN has best values in low Pre-SNR, while the PU-RLS algorithm has the best values in high pre-SNR, the bold font represents the best values, while the red ones represent the least and the worst values.

Table 2
Performance contrast of various algorithms for BW removal

Algorithm / validator	Low Pre-SNR = -5.42 dB, high Pre-SNR = 5 dB Post-SNR dB	MSE	PRD%
PU-LMS	18.244, 23.335	0.0223, 0.0134	40.1639, 31.134
PU-RLS	30.7817, **26.4619**	0.0062, **0.0098**	21.4578, **26.631**
PU-NN	**76.3047**, 25.8296	**4.9×10-5,** 0.0104	**2.2033**, 27.4864

C. EM reduction

In this noise case, we also evaluate the different algorithms under two pre-SNR dB: one low SNR=-10.51 dB and one high SNR =5 dB. In the case of high SNR dB, we take the EM as the reference input to adaptive filter $x(n)$ where $x_1(n)$ represents the primary input to adaptive filter as shown in Figure 9, and we mention only the results of the algorithms.

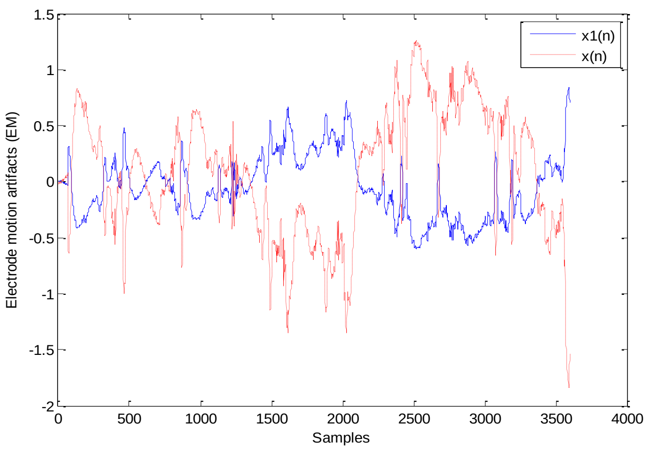

Figure 9: The primary and reference noises for EM

Table 3 shows the results of the performance contrast of various algorithms for removing EM from ECG signal at 300 iterations. Partial update on the neural network has the best values for validation metrics in low pre-SNR, although the RLS algorithm is the best for high pre-SNR with bold lines for best values and underlines and red values for the worst values.

Table 3
Performance contrast of various algorithms for EM removal

Algorithm/ validator	Low Pre-SNR = -10.51 dB, High Pre-SNR= 5 dB Post-SNR dB	MSE	PRD%
PU-LMS	18.9601, 30.1859	0.0206, 0.0067	38.7513, 22.0979
PU-RLS	32.2517, **38.6997**	0.0054, **0.0029**	19.9371, **14.4370**
PU-NN	**106.91**, 36.7375	**1.93×10-6,** 0.0035	**0.4769,** 15.9315

D. MA reduction

In this noise case, the different algorithms are also evaluated under two pre-SNR dB: one low SNR dB=-4.597 and one high SNR dB=5. In the case of high SNR dB, the MA is taken as the reference input to adaptive filter $x(n)$ where $x_1(n)$ represents the primary input to adaptive filter as shown in Figure 10. Only the results of the algorithms are mentioned.

The three algorithms PU_NN, LMS and RLS are introduced and implemented to infer the performance of these techniques to remove MA noise

Table 4 shows the results of the performance contrast of various algorithms for removing MA from the ECG signal at 300 iterations. PU_NN has the best values for three metrics for low Pre-SNR dB= -4.597, Post-SNR dB greater than PU_RLS three times, also PRD% and MSE values are lower in PU_NN relative to both PU_MSE and PU_RLS.

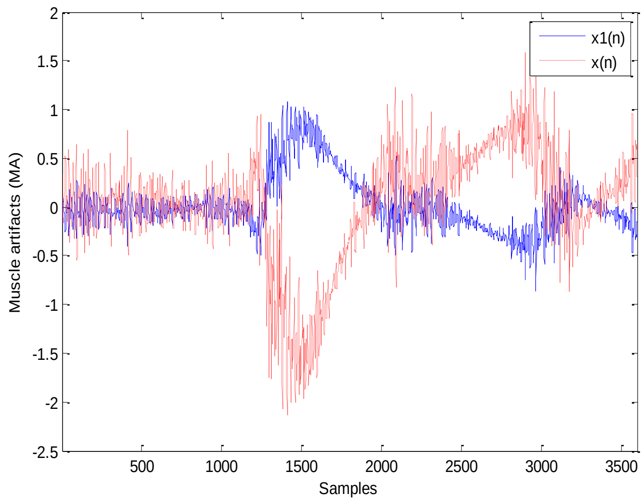

Figure 10: The primary and reference noises for MA

For Pre-SNR high dB=5, all values for three metrics are the best values in PU-RLS.

Table 4
Performance contrast of various algorithms for MA removal

Algorithm/ validator	Pre-SNR, Low = -4.597 dB, High = 5 dB		
	Post-SNR dB	MSE	PRD%
PU-LMS	22.9445, 28.0345	0.0139, 0.0084	31.7516, 24.6090
PU-RLS	35.5913, **35.2258**	0.0015, **0.0041**	16.8711, **17.1765**
PU-NN	**115.92**, 28.4676	**1.28×10^{-6}**, 0.00802	**0.304**, 24.0898

IV. CONCLUSION

It is important to enhance the performance of ECG signals for both ECG signals acquired to be processed by microprocessor systems or FPGA DSP systems. All are essential for diagnosis purposes in not only ECG systems, but also medical monitors, ECG halters, Telecardiology systems. This work aims to introduce and innovate the impact of partial update on neural network filters and compare this augmentation with other adaptive filters.

The proposed augmentation idea of partial update for NNs algorithms is introduced. It achieves the best values with respect to the three validators or metrics for all low dB range of all noises of ECG signals. These measurements are introduced and implemented for all four noises.

The simulation results for denoising ECG signals of all noises show that the PU_LMS algorithm has slow convergence for all noises for low dB pre-SNR ranges or even for high dB, while PU_RLS has the best values of three metrics or validators in high pre-SNR dB.

The augmentation of partial update is done to NNs algorithms. PU_NN is applied for PLI, BW, EM and MA noises. It has drawback only in high dB range for BW, EM and MA noises, while PU_RLS achieves best values. It can be added by improving NN with variable step sizes to overcome the limitations in high dB range before the PU augmentation.

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