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Simplified a Posteriori Probability Calculation for Binary LDPC Codes

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ABSTRACT

The LDPC (Low-Density Parity-Check) decoder must operate soft decisions calculated using: LLR (Log-Likelihood Ratio) or APP (A Posteriori Probability) according to the decoding algorithm used. The exact calculation of these decisions for high order constellations involves complicated operations. In this work, a method to simplify the APP calculation is introduced. It is programmed to adapt as perfectly as possible the transmission system to the channel type in question. This method leads to simplify the implementation of the transmission system. Simulation results show that, under the Gaussian channel, the simplified APP algorithm for 16-QAM achieves the same performance that obtained with the exact APP, while for 64-QAM we have a small performance degradation. The same simplified APP algorithm that used for the Gaussian channel can be applied, with minor operations added, for Rayleigh channel, and it shows a small performance loss with respect to the exact APP.

Keywords: A Priori Probability (APP), Binary LDPC code, Gaussian channel, Log-Likelihood Ratio (LLR), Rayleigh channel, Square-QAM-Gray mapping

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INTRODUCTION

Given the increasing number of applications require high-speed transmission without increasing the bandwidth of the transmission channel, this is the reason for the use of high order constellations. The Quadrature Amplitude Modulation (QAM), is highly recommended as a high order constellation. However, communication systems using QAM require a high signal to noise ratio.

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To overcome this disadvantage, it is interesting to combine high error correction codes such as LDPC codes with QAM.

LDPC codes are linear block codes, can approach the Shannon limit (Shannon, 1948). They were proposed by Gallager in 1962 (Gallager, 1962 & Gallager, 1963) and rediscovered by Mackay in 1995 (MacKay, 1999 & Davey et al., 2002). Their decoding is done according to the principle of iterative decoding. One class of algorithms used to decode LDPC codes is commonly known message propagation algorithms (Johnson, 2010).

Message propagation algorithms are also known as iterative decoding algorithms. The first practical iterative decoding algorithm is the Sum-Product algorithm (SPA) (Gallager, 1963), also known as the belief propagation algorithm is an optimal iterative decoding algorithm but with a high computational complexity. Several algorithms have been proposed to reduce the complexity of the SPA. The messages exchanged in the SPA and its versions can be measured by the APP or the LLR depending on the type of algorithm. Therefore, the message at the decoder input must be using the same calculation. The calculation of the input messages is done to the channel output depends on the considered constellation. However, the number of operations performed by the QAM, to do the calculation of soft decisions, introduced to the decoder, increases with the constellation order. The calculation of soft decisions can be calculated using the LLR or the APP depending on the type of considered algorithm.

Several algorithms have been introduced in order to simplify the exact calculation of the LLR. The pragmatic algorithm, introduced in, attempts to simplify the calculation assuming that the likelihood values are Gaussian variables (LeGoff et al., 1994; LeGoff, 2000). The max-log-MAP algorithm is the most popular simplifying the exact algorithm (Liu et al., 2015). However, simplifications of LLR are used only for decoders based on LLR. While for decoders based on APP, we need to calculate the APP, and the latter is complex.

This work introduces a method to simplify the APP calculation. It is programmed to adapt as perfectly as possible the transmission system to the channel type. This method leads to simplify the implementation of the transmission system. In this paper, we restrict our description of combining binary LDPC code, decoded by FFT-SPA (Barnault et al., 2003 & Carrasco et al., 2008) that uses APP calculations, with square Gary-QAM constellations, over Gaussian and Rayleigh Channels. The rest of the paper is organized as follows: In section 2, a system combining a QAM and an LDPC code is presented, In section 3, the exact APP computation is given. The simplified APP computation, under Gaussian and Rayleigh channels, for square QAM constellations is presented in Section 4. Finally, the simulation results and concluding remarks are given in Sections 5 and 6, respectively.

System Combining a QAM and An LDPC Code

2^m-QAM uses a set of 2^m signals of duration T transmit m symbols $\{u_{n,i}\}$, $i \in \{1, ... , m\}$. From a theoretical view point, the modulation operation at the instant nT is to represent, in a two-dimensional space, the 2^m signals by a set of 2^m points called constellation, and to correspond the set of m symbols to each point of the constellation identified by its abscissa a_n and ordinate b_n.

So each set of m binary symbols is associated, at each time nT, to a couple of symbols (a_n, b_n). After passing through the transmission channel, the observation relating to the couple (a_n, b_n) is represented by a couple (a'_n, b'_n). The transmitted symbols are better follow a Gray mapping, it allows to affirm that there is usually only one erroneous symbol (Tan et al., 2014). The simplest diagram of a digital transmission system as part of the association of an LDPC code and a 2^m-QAM, is given in Figure 1.

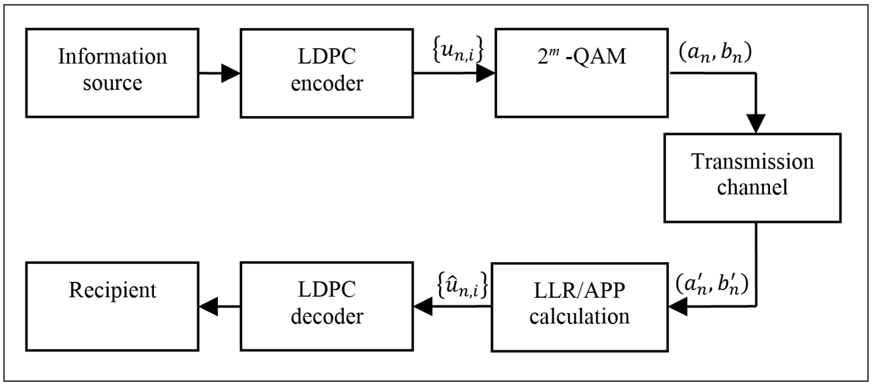

Figure 1. Diagram of a digital transmission system

At the reception, we treat couples (a'_n, b'_n) representative of the couple (a_n, b_n) to extract m samples $\{\hat{u}_{n,i}\}$, $i \in \{1, ... , m\}$ each representative of a binary symbol $u_{n,i}$ associated with the same signal during the modulation. Whatever the constellation with 2^m states used, for each couple (a'_n, b'_n) received at instant nT, the sample $\hat{u}_{n,i}$ is obtained using two relationships, $LLR(u_{n,i})$ (Log-Likelihood Ratio) or $APP(u_{n,i})$ (A Posteriori Probability):

$APP(u_{n,i})$, $i \in \{1, \dots, m\}$, is calculated as follows (Moon, 2005):

$$APP(u_{n,i} = 0) = \frac{Pr\{(a'_n, b'_n) / u_{n,i}=0\}}{Pr\{(a'_n, b'_n) / u_{n,i}=0\} + Pr\{(a'_n, b'_n) / u_{n,i}=1\}} \quad (1.a)$$

$$APP(u_{n,i} = 1) = 1 - APP(u_{n,i} = 0) \quad (1.b)$$

Where $\Pr\{(a'_n, b'_n) / u_{n,i} - w\}$ is the probability that the available couple is (a'_n, b'_n); knowing the binary symbol $u_{n,i}$ is equal to w.

$LLR(u_{n,i})$, $i \in \{1, \dots, m\}$, is calculated as follows (Alam et al., 2009):

$$LLR(u_{n,i}) = log\left[\frac{Pr\{(a'_n, b'_n) / u_{n,i} = 1\}}{Pr\{(a'_n, b'_n) / u_{n,i} = 0\}}\right] \quad (2)$$

Equation (2) is the exact calculation of LLR, it is the optimal calculation that represents the log-MAP algorithm (Maximum A Posteriori) (Tosato et al., 2002, Hyun et al., 2005; Wang et al., 2011). However, it involves complicated operations. Several algorithms have been introduced in order to simplify the exact calculation of the LLR.

In this work, we propose a method for applying the simplified calculation of LLR and adapt them to the LDPC decoder based on the APP, and even to simplify the APP calculation. This provided to insert an additional module to make the conversion LLR to APP, as shown in figure 2. The simplified algorithm of the APP got on the Gaussian channel can be reused efficiently on a Rayleigh channel (Figure 2), this provided insert an additional operation to accommodate, each time nT, the channel attenuation a_n.

In Figure 2, the block of the simplified calculation of the LLR is identical for all decoding algorithms, whatever the decoder input: LLR or APP. Only the modulus of the conversion of APP to LLR depends on the decoder input. His presence in the figure 2, is justified by the need to apply the simplified calculation of the LLR to LDPC decoder based on APP. Their presence is not essential in the case of an LDPC decoder based on LLR. Indeed, it is easy to change a decoding algorithm based on LLR to an algorithm based on APP, keeping unchanged the simplified calculation of LLR.

Figure 2 shows that the proposed method can simplify the implementation of the system. In what follows, we will show that this method simplifies the exact calculation of APP. The method will be explained for a QAM with 2^{2p} states, $p \in \mathbb{N}$.

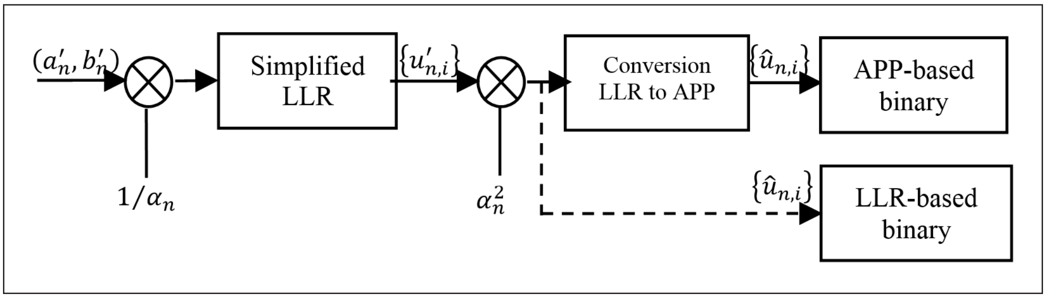

Figure 2. Principle of simplified APP calculation

Exact Calculation of APP

2^{2p}-QAM, uses a square constellation, has the particularity to be reduced to two Amplitude Modulations with 2^p states independently acting on two carriers in phase and quadrature (Ghaffar et al., 2010). According to the above property (the case of a square constellation), p expressions in phase, obtained from the equation (2) are consequently the following:

$$APP(u_{n,i}=0)=\frac{Pr\{a'_n/u_{n,i}=0\}}{Pr\{a'_n/u_{n,i}=0\}+Pr\{a'_n/u_{n,i}=1\}},\ i\in\{1,\dots,p\} \quad (3.a)$$

$$APP(u_{n,i}=1)=1-APP(u_{n,i}=0),\ i\in\{1,\dots,p\} \quad (3.b)$$

And p expressions in quadrature, obtained from the equation (2), are the following:

$$APP(u_{n,i}=0)=\frac{Pr\{b'_n/u_{n,i}=0\}}{Pr\{b'_n/u_{n,i}=0\}+Pr\{b'_n/u_{n,i}=1\}},\ i\in\{p+1,\dots,2p\} \quad (4.a)$$

$$APP(u_{n,i}=1)=1-APP(u_{n,i}=0),\ i\in\{p+1,\dots,2p\} \quad (4.b)$$

For a Gaussian transmission channel, with the noise variance σ^2, the p relations in phase eventually lead to the following expressions:

$$APP(u_{n,i}=0)=\frac{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a'_n-a^0_{i,j}\right)^2\right\}}{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a'_n-a^0_{i,j}\right)^2\right\}+\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a'_n-a^1_{i,j}\right)^2\right\}},$$
$$i\in\{1,\dots,p\} \quad (5.a)$$

$$APP(u_{n,i}=1)=1-APP(u_{n,i}=0),\ i\in\{1,\dots,p\} \quad (5.b)$$

Where $a^k_{i,j}$ are possible values of the symbol a_n when the symbol $u_{n,i}$ to be transmitted has the value k ($k = 0$ or 1).

Similarly, for a Gaussian channel, the p relations in the quadrature path eventually lead to the following expressions:

$$APP(u_{n,i}=0)=\frac{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(b'_n-b^0_{i,j}\right)^2\right\}}{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(b'_n-b^0_{i,j}\right)^2\right\}+\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(b'_n-b^1_{i,j}\right)^2\right\}},$$
$$,i\in\{p+1,\dots,2p\} \quad (6.a)$$

$$APP(u_{n,i}=1)=1-APP(u_{n,i}=0),\ i\in\{p+1,\dots,2p\} \quad (6.b)$$

Where $b^k_{i,j}$ are possible values of the symbol b_n when the symbol $u_{n,i}$ to be transmitted has the value k ($k = 0$ or 1).

Equations (5) and (6), representing the exact calculation of APP, reflects full information for all possible QAM symbols. Therefore, it is necessary a large number of calculations to calculate the exact APP in the case of a high order constellation such as 16-QAM, 64-QAM, 256-QAM. Thereafter, we will simplify the calculation, using the proposed method, when the channel is Gaussian. Then, we will show that simplified equations obtained on the Gaussian channel can be effectively reused on a Rayleigh channel.

Simplified Calculation of App

Gaussian Channel. Following the figure (3), simplification of equations (5) and (6) is obtained after the simplified calculation of LLR then converting the LLR to the APP. As the transmission channel is Gaussian, and the modulation uses a square constellation, the calculation of LLR obtained from equation (2) are well approached, using two algorithms max-log-MAP and pragmatic.

Max-log-MAP Algorithm. The Max-log-MAP algorithm, introduced in (Liu et al., 2015) shows that the p relations in the phase and p relations in the quadrature are given respectively by equation (7.a) and equation (7.b):

$$LLR(u_{n,i}) = \frac{\left(\min\limits_{j\in\{1,\dots,2^{p-1}\}}\left(a'_n - a^0_{i,j}\right)\right)^2 - \left(\min\limits_{j\in\{1,\dots,2^{p-1}\}}\left(a'_n - a^1_{i,j}\right)\right)^2}{2\sigma^2},\ i \in \{1,\dots,p\} \quad (7.a)$$

$$LLR(u_{n,i}) = \frac{\left(\min\limits_{j\in\{1,\dots,2^{p-1}\}}\left(b'_n - b^0_{i,j}\right)\right)^2 - \left(\min\limits_{j\in\{1,\dots,2^{p-1}\}}\left(b - b^1_{i,j}\right)\right)^2}{2\sigma^2},$$

$$i \in \{p+1,\dots,2\,p\} \quad (7.b)$$

Pragmatic Algorithm. The pragmatic algorithm introduced in (LeGoff et al., 1994) shows that the p relations in the phase and p relations in the quadrature multiplied by $\sigma^2/2$, are given respectively by equation (8.a) and equation (8.b):

$$LLR(u_{n,1}) = a'_n$$

$$LLR(u_{n,2}) = -\left|LLR(u_{n,1})\right| + 2^{p-1}$$

$$LLR(u_{n,i}) = -\left|LLR(u_{n,i-1})\right| + 2^{p-i+1} \quad (8.a)$$

$$LLR(u_{n,p}) = -\left|LLR(u_{n,p-1})\right| + 2$$

And

$$LLR(u_{n,p+1}) = b'_n$$

$$LLR(u_{n,p+2}) = -\left|LLR(u_{n,p+1})\right| + 2^{p-1}$$

$$LLR(u_{n,p+i}) = -|LLR(u_{n,p+i-1})| + 2^{p-i+1}$$
$$LLR(u_{n,2p}) = -|LLR(u_{n,2p-1})| + 2 \quad (8.b)$$

For a good approximation, we multiply equations (8) with a constant factor $f = 2 / \sigma^2$, we get:

$$LLR(u_{n,1}) = f \times a'_n$$
$$LLR(u_{n,2}) = f \times (-|LLR(u_{n,1})| + 2^{p-1})$$
$$LLR(u_{n,i}) = f \times (-|LLR(u_{n,i-1})| + 2^{p-i+1}) \quad (9.a)$$
$$LLR(u_{n,p}) = f \times (-|LLR(u_{n,p-1})| + 2)$$

And

$$LLR(u_{n,p+1}) = f \times b'_n$$
$$LLR(u_{n,p+2}) = f \times (-|LLR(u_{n,p+1})| + 2^{p-1})$$
$$LLR(u_{n,p+i}) = f \times (-|LLR(u_{n,p+i-1})| + 2^{p-i+1}) \quad (9.b)$$
$$LLR(u_{n,2p}) = f \times (-|LLR(u_{n,2p-1})| + 2)$$

The derivation of APP from the simplified LLR (Lee et al., 2005), leads to the following simplified equations:

$$APP(u_{n,i} = 0) = \frac{1}{1+exp(LLR(u_{n,i}))},\ i \in \{1, \ldots, 2p\} \quad (10.a)$$

$$APP(u_{n,i} = 1) = 1 - APP(u_{n,i} = 0),\ \{1, \ldots, 2p\} \quad (10.b)$$

So it is remarkable that the simplified calculation of the APP, equations (10), are less number of calculations that the exact calculation of the APP, equations (5) and (6). Illustratively, consider for example the case of 16-QAM (p = 2), the functions relating the inputs and outputs, using both algorithms, pragmatic and max-log-MAP, are shown in figures 4. In each of Figures 3, there is shown the function whose form is given by equations (5) and (6) and the corresponding simplified function, equations (10), this for a signal to noise equals to 4 dB.

For bits $u_{k,3}$ and $u_{k,4}$ we have the same results as presented in figure 3.a and figure 3.b respectively. In all cases, these curves are used to verify that, despite their simplicity, the approximations performed by equations (10) are excellent, and they can therefore be used advantageously in place of expressions (5) and (6).

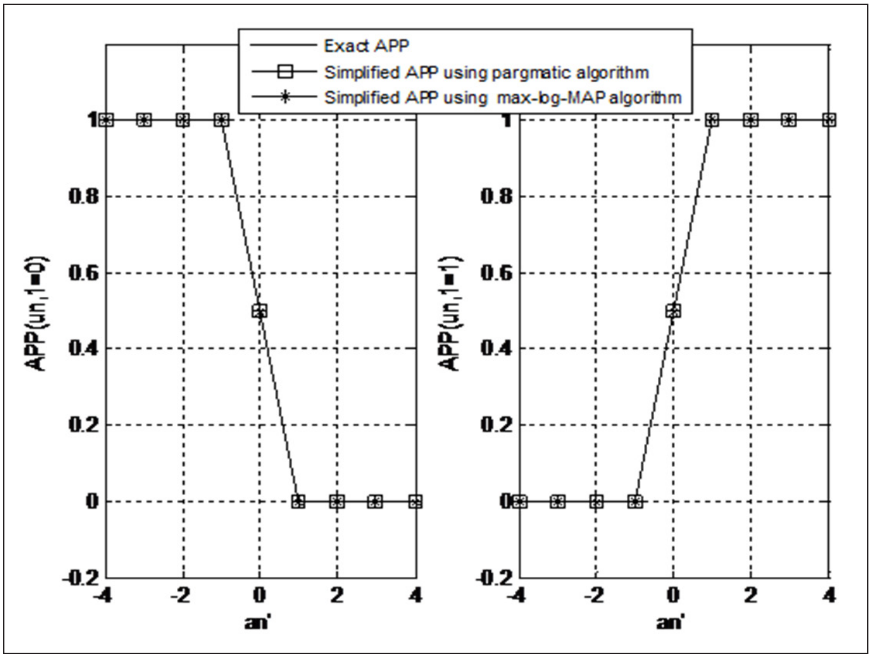

Figure 3a. Exact and simplified APP corresponding to bit $u_{n,1}$ for a 16-QAM constellation, at $E_b/N_0 =$ 4dB

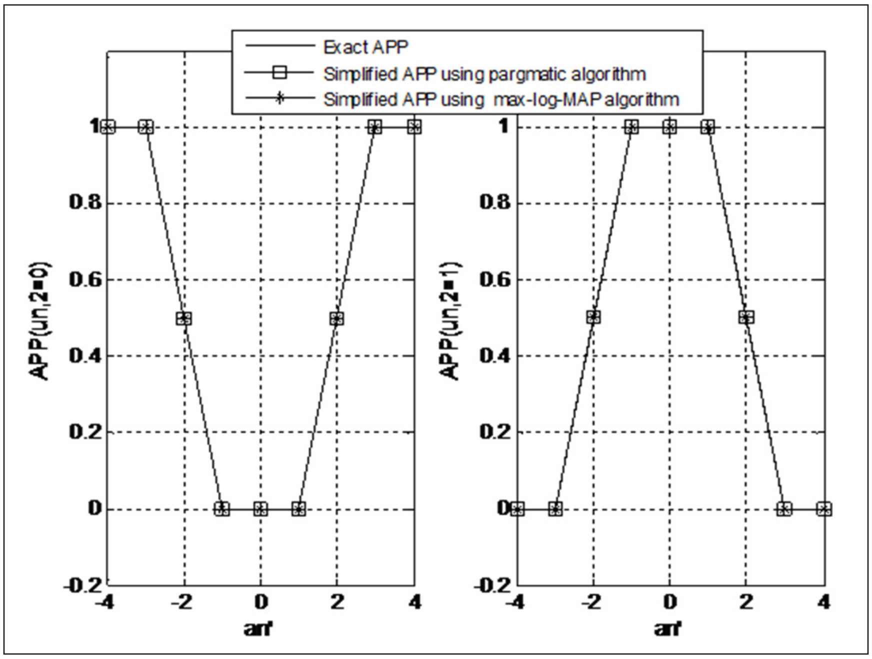

Figure 3b. Exact and simplified APP corresponding to bit $u_{n,2}$ for a 16-QAM constellation, at $E_b/N_0 =$ 4dB

Rayleigh Channel. In this section, it is assumed that the transmission channel is a Rayleigh channel. In addition, it is assumed that the attenuation of the channel a_n, at time nT, is known perfectly by the receiver (Zehevi, 1992). We will show that it is not necessary to repeat the whole argument made about the Gaussian channel to get algorithms that allow to implement the simplifications made on a Rayleigh channel (LeGoff, 1995). First, recall that the channel output at time nT, the information available relative to the couple (a_n, b_n) is such that:

$$a'_n = \alpha_n a_n + z_n \quad (11.a)$$

$$b'_n = \alpha_n b_n + z_n \quad (11.b)$$

Where z_n is a Gaussian noise, centered, with variance σ^2 and a_n variable characterizes the attenuation of the transmitted signal. As the variable a_n at time nT is known, it is possible to divide the two samples a'_n and b'_n available at the output of the channel by a_n (LeGoff, 2000). Samples a''_n and b''_n thus obtained are expressed in the form:

$$a''_n = \frac{a'_n}{\alpha_n} = a_n + z'_n \quad (12.a)$$

$$b''_n = \frac{b'_n}{\alpha_n} = b_n + z'_n \quad (12.b)$$

Where z'_n is a Gaussian noise, centered, with variance σ^2_n equals to σ^2 / a^2. Since the samples a''_n and b''_n are modeled by Gaussian variables, it is possible to apply directly on the samples a''_n and b''_n available, simplified algorithms of LLR strictly identical to those used when the transmission channel is Gaussian, and this irrespective of the modulation used. In order to apply best these results obtained with a modulation to a proposed system in this work, so it is necessary to multiply by a^2_n the samples $u'_{n,i}$ - $LLR(u_{n,i})$, before the conversion (Figure 2).

RESULTS

In this section, we will show the effect of simplifying calculation of the APP on the performance of binary LDPC code of rate equals to 1/2 and a parity check matrix of size 128 × 256, and a decoding algorithm using the FFT-SPA where the number of iterations is four. The transmission chain for which we evaluated the Bit Error Rate (BER) after decoding is one that has been shown in Figure 1, for a binary LDPC code, attached to two square constellations: 16-QAM and 64-QAM, using Gray mapping, and two transmission channels: Gaussian and Rayleigh.

Figure 4 shows performance comparisons, on a Gaussian channel, between a binary LDPC code using the exact calculation of the APP, equations (5) and (6), a binary LDPC code using the simplified calculation (equations (10)) by applying the pragmatic algorithm (equations (9)) and a binary LDPC code using the simplified calculation (equations (10)) by applying the max-log algorithm (equation (7)).

In the figure 4 we can see that the simplified calculation of the APP for 16-QAM,on a Gaussian channel, using a max-log-MAP algorithm and a pragmatic algorithm, has no effect on the performance of a binary LDPC code. While, with 64-QAM we have a few performance degradation when we use a pragmatic algorithm to simplify the APP calculation.

In order to study the influence of the simplified calculation on the performance of a binary LDPC code on a Rayleigh channel, the same performance comparison of Figure 4 are performed on a Rayleigh channel, in Figure 5.

Seen the results on a Rayleigh channel, there is a small performance degradation of a binary LDPC code using the pragmatic and max-log-MAP algorithms to simplify the APP.

Figure 6 presents the same performance comparison of Figures 4 and 5, using 16-QAM, on a Gaussian and Rayleigh channels, where the number of iterations is six. As illustrated in figure 6, the proposed simplified calculations are also good for binary LDPC codes with six iterations.

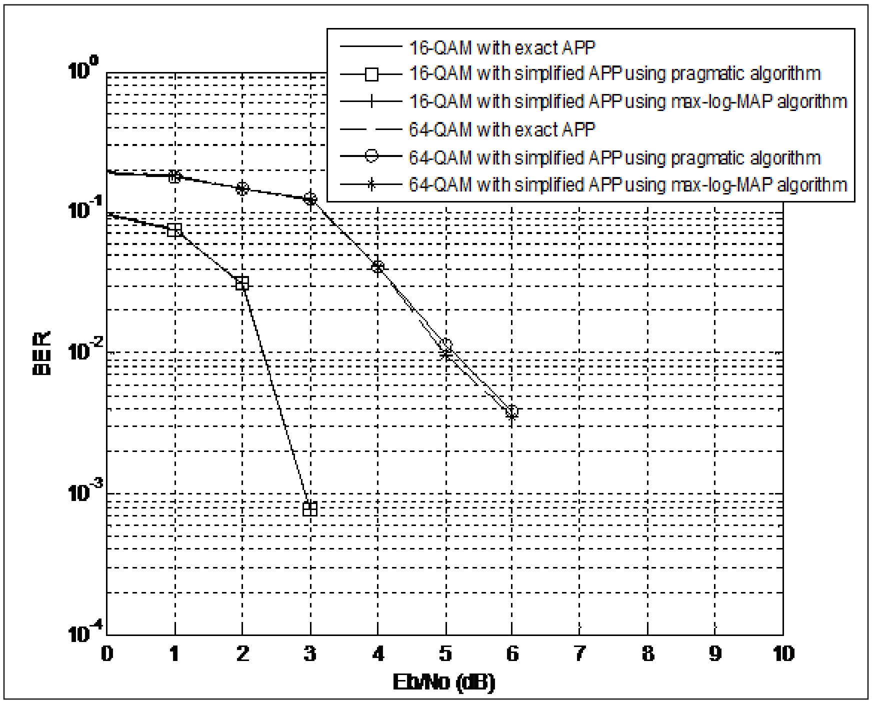

Figure 4. Performance comparisons, under Gaussian channel, of binary LDPC code decoded by FFT-SPA using exact APP and simplified APP algorithms of 16-QAM and 64-QAM constellations

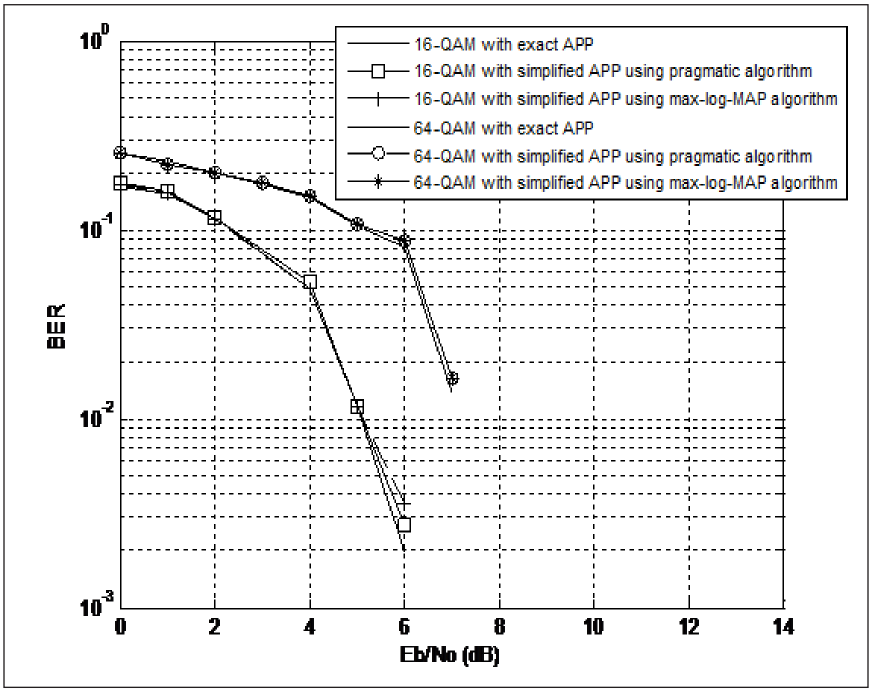

Figure 5. Performance comparisons, under Rayleigh channel, of binary LDPC code decoded by FFT-SPA using exact APP and simplified APP algorithms of 16-QAM and 64-QAM constellations

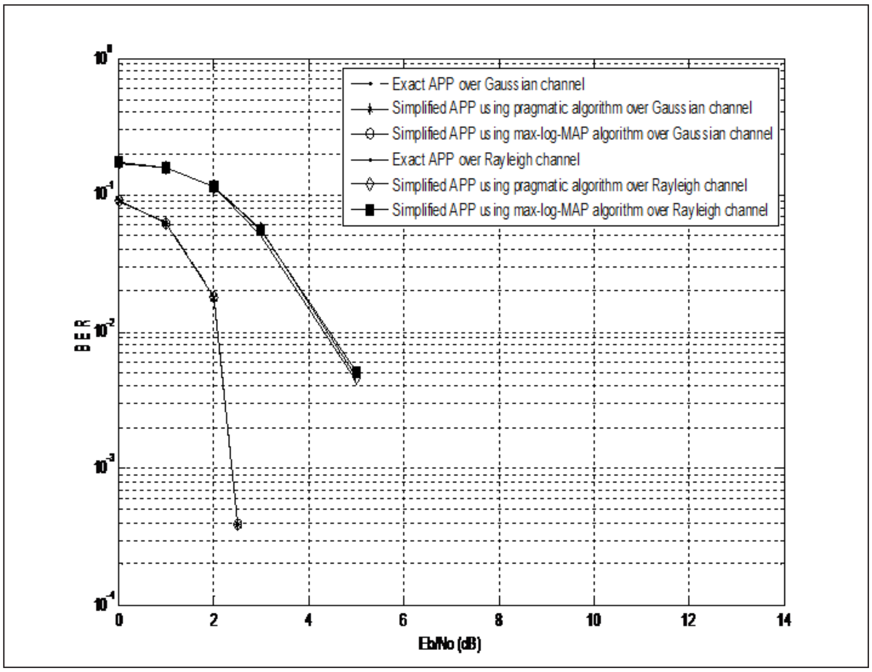

Figure 6. Performance comparisons, under Gaussian and Rayleigh channels, of binary LDPC code decoded by FFT-SPA using exact APP and simplified APP algorithms of 16-QAM with six iterations

Therefore, the simulation results presented in Figures 4, 5 and 6 show that the binary LDPC code using the proposed simplified computations can be used effectively in a system with high spectral efficiency, i. e. with high order constellations.

CONCLUSION

In this work, we used the simplified calculation of the LLR to facilitate the calculation of the APP for binary LDPC codes. The proposed method for making these simplification, puts a system combining a binary LDPC code and a high order constellation simple to implement. It is programmed to adapt as perfectly as possible the system to the type of channel in question. Also, it ensures an efficient decoding regardless of the channel in type. Since LDPC codes are selected as candidate for the 5th generation wireless communications (5G), it is interesting to use the proposed simplifications.

REFERENCES

Alam, M. Z., Islam, C. S., & Sobhan, M. A. (2009). Using Log Likelihood Relation for BER Analysis of QAM in Space Diversity. *Journal of Communications, 4*(6), 371-379.

Barnault, L., & Declercq, D. (2003). Fast Decoding Algorithm for LDPC over GF(2*q)*. In *Proceeding IEEE Information Theory Workshop* (pp. 70-73). Paris, France.

Carrasco, R. A., & Johnston, M. (2008). *Non-Binary Error Control Coding for Wireless Communication And Data Storage*. Singapore: Wiley & Sons (Asia) Pte Ltd.

Davey, M., & MacKay, D. (2002). Low-Density Parity Check Codes over GF(*q*). *IEEE Communications Letters, 2*(6), 165–167.

Gallager, R. (1962). Low-Density Parity-Check Codes. *IEEE Transaction Information Theory, 8*(1), 21–28.

Gallager, R. G. (1963). *Low-density parity-check codes* (Ph.D. dissertation). Department of Electrical Engineering, M.I.T., Cambridge, Mass.

Ghaffar, R., & Knopp, R. (2010). Low Complexity Metrics for BICM SISO and MIMO Systems. In *2010 IEEE 71st Vehicular Technology Conference (VTC 2010-Spring)* (pp. 1–6). Taipei, Taiwan.

Hyun, K., & Yoon, D. (2005). Bit Metric Generation for Gray Coded QAM Signals. *IEE Proceedings-Communications, 152*(6), 1134–1138.

Johnson, S. (2010). *Iterative Error Correction Turbo, Low-Density Parity-Check And Repeat-Accumulate Codes*. United States of America: Cambridge University Press.

Lee, S. H., Seok, J. A., & Joo, E. K. (2005). Serially Concatenated LDPC and Turbo Code with Global Iteration. In *Proceedings of the 18th International Conference on Systems Engineering (ISCEng'05)* (pp. 201-204). Las Vegas, NV, USA.

LeGoff, S. (1995). *Les Turbo-Codes et leurs Applications aux Transmissions à Forte Efficacité Spectrale*. PhD PhD Thesis. University of Western Brittany, Brest.

LeGoff, S. (2000). Bit-Interleaved Turbo-Coded Modulations for Mobile Communications. In *European Signal Processing Conference (EUSIPCO)* (pp. 1-4). Finlande.

LeGoff, S., Berrou, C., & Glavieux, A. (1994). Turbo-Codes and High Spectral Efficiency Modulations. In *IEEE International Conference on Communications (ICC'94)* (pp. 645-649). New Orleans.

Liu, X., & Kosakowski, M. (2015). Max-Log-MAP Soft Demapper with Logarithmic Complexity for M-PAM Signals. *IEEE Signal Processing Letters, 22*(1), 50-53.

MacKay, D. (1999). Good Error-Correcting Codes Based on Very Sparse Matrices. *IEEE Transaction Information Theory, 45*(2), 399–431.

Moon, T. K. (2005). *Error Correction Coding Mathematical Methods and Algorithms*. Hoboken: John Wiley & Sons, Inc.

Shannon, C. E. (1948). A Mathematical Theory of Communication. *Bell System Technical Journal, 27*, 379-423.

Tan, J., Wang, Q., Qian, C., Wang, Z., Chen, S., & Hanzo, L. (2014). A Reduced-Complexity Demapping Algorithm for BICM-ID Systems. *IEEE Transactions on Vehicular Technology, 64*(9), 4350-4356.

Tosato, F., & Bisaglia, P. (2002). Simplified Soft-Output Demapper for Binary Interleaved COFDM with Application to HIPERLAN/2. In *IEEE International Conference Communication* (Vol. 2, pp. 664–668). New York, USA.

Wang, L., Xu, D., & Zhang, X. (2011). Low Complexity Bit Metric Generation for PAM Signals Based on Non-Linear Function. *Electronics Letters, 47*(17), 966–967.

Zehevi, E. (1992). 8-PSK trellis codes for a Raleigh Channel. *IEEE Transaction on Communications, 40*(5), 873-884.