Journal of Engineering Science and Technology
Special Issue on ISSC'2016, April (2017) 158 - 168
© School of Engineering, Taylor's University

SOLVING THE SINGLE-PERIOD INVENTORY ROUTING PROBLEM WITH THE DETERMINISTIC APPROACH

MOHD KAMARUL IRWAN ABDUL RAHIM*, SANTHIRASEGARAN S. R. NADARAJAN, MOHD AKHIR AHMAD

School of Technology Management and Logistics, Universiti Utara Malaysia,
06010 UUM Sintok, Kedah, Malaysia.
*Corresponding Author: mk.irwan@uum.edu.my

Abstract

This paper considers the problem of managing inventory and routing problems in a two-stage supply chain system under a Vendor Managed Inventory (VMI) policy. VMI policy is an integrating decisions between a supplier and the customers. Thus, we assumed that the demand at each customer is stationary and the warehouse is implementing a VMI. The focus of this research is to optimize the inventory and the transportation costs of the customers for a two-stage supply chain system. The problem is to define the delivery quantities, delivery times and routes to the customers for the single-period deterministic inventory routing problem (SP-DIRP) system. As a result, a linear mixed-integer program is established for the solutions of the SP-DIRP problem. A detailed analysis of an illustrative example for solving the SP-DIRP problem is presented to describe the integration of the proposed model.

Keywords: Single-period, Deterministic model, Inventory routing problem, Supply chain optimization, Vendor managed inventory.

1. Introduction

Vendor Managed Inventory (VMI) is an integrated inventory management policy in which the supplier assumes the responsibility of maintaining the inventory at the customers while ensuring that they will not run out of stock-outs. The delivery times to the customers are no longer carried in response to customers' orders; instead the supplier chooses when each delivery takes place. In this way, the planning is proactive as it is based on the available information rather than reactive to customers' orders. Therefore, this policy has many benefits for both the supplier and the customers. For instance, the supplier can combine multiple deliveries to optimize the routing and the fleet loading. Moreover, the deliveries also become more uniform, fewer inventories need to be held at the warehouse. Under the VMI policy, the customer has no longer to dedicate resources to inventory management. Also the service level increases, as the supplier exactly knows the inventory level at the customers and can determine the precise urgency for each.

In this paper, we developed a model for the solution of customers' demands on routing and inventory management in a two-stage supply network. The main objective in this paper is to optimize the overall inventory and transportation costs while assuming that the demand at each customer is known and deterministic. Therefore, we are focussed with a single-period deterministic inventory routing problem (SP-DIRP) where the customers consume some product at a stationary demand rate in a single-period time. More specifically, we study a distribution system in which a fleet of homogeneous vehicles is used to assign the product from a single warehouse to a set of customers. Based on the formulation of the single-period IRP [1,2] and multi-period IRP by [3], we formulated a linear mixed-integer model for the SP-DIRP.

The remainder of this paper is organized as follows. In section 2, we review some papers on the modelling of deterministic IRP (DIRP). In Section 3, we develop a linear mixed-integer formulation for the SP-DIRP. A detailed analysis of an illustrative supply chain example is given in Section 4. Finally, we provide some concluding remarks in Section 5.

2. Literature Review

Inventory Routing Problems (IRP) is an important optimization model that captures the essential characteristics of VMI agreements such as inventory control and transportation scheduling. The IRP is a very challenging problem that arises in various distribution systems. It involves managing simultaneously inventory control and vehicle routing in organizations where one or several warehouses are responsible for the replenishment of a set of geographically dispersed customers.

Many versions of the deterministic IRP have been studied, to investigate the integrated inventory management and vehicle routing. Table 1 shows the classification of main literature on the deterministic IRP, which is divided into finite and infinite planning horizon problems. However, in this section, we just focused on the deterministic IRP literature with the finite planning horizon.

Dror et al. [4] are among the earliest paper to address the IRP, propose a short term solution approach to take into account what happens after the single day planning period. They described this problem over a short planning period, e.g. one week, and proposed a mixed integer programming model where effects of present decisions on later periods. Bertazzi et al. [5] address a multi-period model with a deterministic case in which a set of products is shipped from a common supplier to several customers. For each product, a starting level of the inventory is given both for the supplier and for each customer, and the level of the inventory at the end of the time horizon can be different from the starting one.

Campbell and Savelsbergh [6] also study a multi-period IRP where their work is motivated by an application in the industrial gases industry, PRAXAIR, which is a large industrial gases company with about 60 production facilities and more than 10,000 customers across North America. The authors propose a two-phase solution approach. In the first phase, they determine which retailers receive a delivery on each day of the planning period and decide on the size of the deliveries. In the second phase, they then determine the actual delivery routes and schedules for each of the days.

Abdelmaguid and Dessouky [7] propose a genetic algorithm (GA) approach for solving the integrated inventory distribution problem in the multiple planning periods, in which backorders are permitted. Backorder decisions are generally established in two cases. In the first case is when there is insufficient vehicle capacity to deliver to a customer, while in the second case is when there is a transportation cost saving that is higher than the incurred backorder cost by a customer. Archetti et al. [8] propose an exact algorithm to the deterministic IRP over a given time horizon. Each customer defines a maximum inventory level. The supplier monitors the inventory of each customer and determines its replenishment policy, guaranteeing that no stock-out occurs at the customer.

Yugang et al. [9] consider a multi-period deterministic inventory routing problem with split delivery (IRPSD) where the customers' demands in each period over a given planning horizon are assumed to be constant and must be satisfied without backorder. The delivery of each customer in each period can be split and performed by multiple vehicles. For recent study on the multi-period deterministic IRP, Archetti et al. [10] consider the IRP in discrete time, where a supplier has to serve a set of retailers over a time horizon. A capacity constraint for the inventory is given for each customer and the service cannot cause any stock-out situation. Two different replenishment policies are considered, the order-up-to level (OU) and the maximum level (ML) policies. To solve the IRP, the authors developed a powerful hybrid heuristic, which operates with a combination of a tabu search embedded within four neighbourhood searches and two mixed integer programming (MIP) models.

Table 1. Classification of main literature on the Deterministic IRP.

Deterministic IRP	
Finite Horizon	**Infinite Horizon**
Dror et al. [4]	Gallego & Simchi-Levi [11]
Bertazzi et al. [5]	Anily & Federgruen [12]
Campbell & Savelsbergh [6]	Bramel & Simchi-Levi [13]
Abdelmaguid & Dessouky [7]	Chan et al. [14]
Archetti et al. [8]	Aghezzaf et al. [15]
Yugang et al. [9]	Jung & Mathur [16]
Archetti et al. [10]	Raa & Aghezzaf [17]

3. Modelling and formulating SP-DIRP

The SP-DIRP involves of a single warehouse using a fleet of homogeneous vehicles to deliver a single product to a set of geographically circulated customers over a given planning horizon. It is assumed that customer-demand rates are stationary, and that travel-times are constant over time. The objective of this SP-DIRP is to determine optimal quantities and delivery time, and vehicle delivery routes, so that to minimize the total inventory and transportation costs. To formulate the model for the SP-DIRP, the following assumptions are made:

- The time required for loading and unloading a vehicle is ignored in the model.
- Inventory capacities at the warehouse and customers are assumed to be large enough so that the capacity constraints are omitted in the model.
- Transportation costs of the vehicles are assumed to be directly proportional to the travel times.
- Split deliveries are not allowed, therefore each customer is always completely replenished by a single vehicle.

Let τ_t be the size in time units of period 1, in this case we are used 8 working hours per day. Let S be the set of customers indexed by i and j; and $S^+ = S \cup \{r\}$, which represents the warehouse. A fleet of vehicles V is used to serve these customers. The parameters and the variables of the model are given below:

- φj : the fixed handling cost per delivery at location $j \in S+$ (customers and warehouse);
- ηj : the per unit per period holding cost of the product at location $j \in S+$;
- ψ v : the fixed operating cost of vehicle $v \in V$;
- δv : travel cost of vehicle $v \in V$;
- κv : the capacity of vehicle $v \in V$;
- νv : average speed of vehicle $v \in V$;
- θij : duration of a trip from customer $i \in S+$ to customer $j \in S+$;
- dj : the stationary demand rate at customer j .
- Ij0 : the initial inventory levels at each customer $j \in S$.
- Q_{ij}^{v}: the quantity of product remaining in vehicle $v \in V$ when it travels directly to location $j \in S+$ from location $i \in S$. This quantity equals zero when the trip (i,j) is not on any tour of the route travelled by vehicle $v \in V$;
- qj : the quantity delivered to location $j \in S$, and 0 otherwise;
- Ij : the inventory level at location (customers and warehouse) $j \in S+$;
- x_{ij}^{v} : a binary variable set to 1 if location $j \in S+$ is visited immediately after location $i \in S+$ by vehicle $v \in V$, and 0 otherwise;
- y^{v} : a binary variable set to 1 if vehicle $v \in V$ is being used, and 0 otherwise.

Thus, if we let I_{jr} be the initial inventory level at the warehouse, the linear mixed-integer formulation for the single-period DIRP is given as follows:

SP-DIRP: Minimize

$$CV = \sum_{v \in V} \left[\psi^v y^v + \sum_{i \in S^+} \sum_{j \in S^+} (\delta_v \nu_v \theta_{ij} + \varphi_j) x_{ij}^v \right] + \sum_{j \in S^+} \eta_j I_j \tag{1}$$

Subject to:

$$\sum_{v\in V}\sum_{i\in S^+} x_{ij}^v \le 1,\ \forall\, j\in S \tag{2}$$

$$\sum_{i\in S^+} x_{ij}^v - \sum_{k\in S^+} x_{jk}^v = 0, \forall\, j\in S^+,\, v\in V \tag{3}$$

$$\sum_{i\in S^+}\sum_{j\in S^+} \theta_{ij} x_{ij}^v \le \tau_t, v\in V \tag{4}$$

$$\sum_{v\in V}\sum_{i\in S^+} Q_{ij}^v - \sum_{v\in V}\sum_{k\in S^+} Q_{jk}^v = q_j,\ \forall\, j\in S \tag{5}$$

$$Q_{ij}^v \le k^v x_{ij}^v, \forall\, j\in S^+,\, v\in V \tag{6}$$

$$I_j + q_j - I_j = d_j \tau_t, \forall\, j\in S \tag{7}$$

$$I_{jo} \le I_j, \forall\, j\in S \tag{8}$$

$$x_{rj}^v \le y^v, \forall\, j\in S^+,\, v\in V \tag{9}$$

$$x_{rj}^v, y^v \in \{0,1\}, I_{j0,} I_j \ge 0, Q_{ij}^v \ge 0, q_j \ge 0,\ \forall\, j\in S^+,\, v\in V$$

Constraints (2) confirm that each customer is visited at most once. Constraints (3) guarantee that if a truck arrives at a customer, it must leave after it has served it to a next customer or to the warehouse. Constraints (4) ensure that trucks complete their routes within one travel period, so the total travel time of a truck should not exceed the total working hours. Constraints (5) determine the quantity delivered to a customer. The truck capacity constraints are given by (6) and assure that the variables Q_{ij}^v, cannot carry any cumulated flow unless x_{ij}^v, equals 1. Constraints (7) are the inventory balance equations at the customers. Constraints (8) indicate that the final inventory level at customer *j* at the end of period is of the same magnitude as its initial inventory. Constraints (9) indicate that a truck cannot be used to serve any customer unless it is selected. The objective function (1) includes four cost components, which are total fixed operating cost of the vehicles, total transportation cost, total delivery handling cost and total inventory holding cost at the warehouse and customers.

4. A detailed analysis of an illustrative example for the SP-DIRP

To illustrate the behaviour of our proposed model, we present some small and medium example case with seven customers and 10 customers for the single-period deterministic inventory routing problem (SP-DIRP). In the first case, we consider only seven customers which are scattered around the warehouse as shown in Fig. 1, and demand rates for each customer d_j are known and generated randomly and uniformly between 0.1 and 3 tons per hour. A fleet of homogeneous vehicles *V* with a capacity of truck κ^v is 30 tons, is available for the distribution of the product. We assume that the truck's average speed v_v is 50 km per hour, and the travel cost δ_v is RM 1 per km. The inventory holding costs η_j for each customer is generated randomly and uniformly between 0.1 and 0.5 (in RM per ton per hour). The fixed operating cost of the vehicle ψ^v is RM 50 per truck. We also assume that the fixed delivery handling cost φ_j is the same for all customers and the size in time unit τ_t is set to 8 hours. The values of these parameters are then displayed in Table 2.

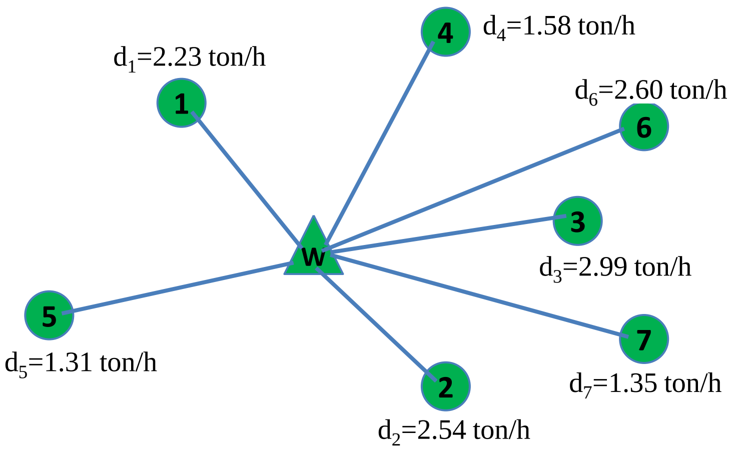

Fig. 1. A small case with 7 customers.

Table 2. Parameters and delivery quantity to the 7 customers for the SP-DIRP.

Customer	Demand (ton/hr)	Inventory holding cost (RM)	Delivery cost (RM)	Delivery quantity (ton)
1	2.23	0.112	100	19.44
2	2.54	0.131	100	30.00
3	2.99	0.117	100	30.00
4	1.58	0.130	100	19.12
5	1.31	0.109	100	10.56
6	2.60	0.144	100	30.00
7	1.35	0.115	100	10.88

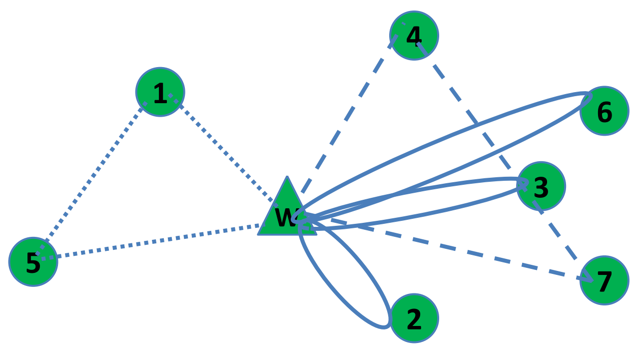

Fig. 2. The optimal solution of the 7 customers for the SP-DIRP.

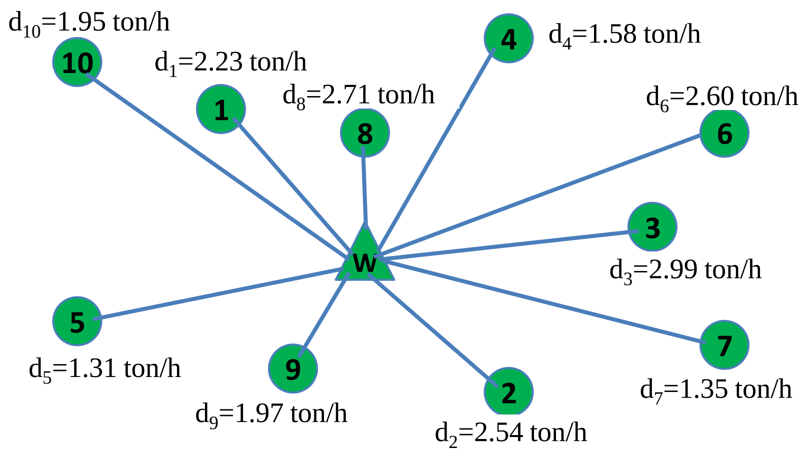

Fig. 3. A medium case with 10 customers.

Table 3. Parameters and delivery quantity to the 10 customers for the SP-DIRP.

Customer	demand (ton/hr)	Inventory holding cost (RM)	Delivery cost (RM)	Delivery quantity (ton)
1	2.23	0.112	100	19.44
2	2.54	0.131	100	30.00
3	2.99	0.117	100	30.00
4	1.58	0.130	100	12.72
5	1.31	0.109	100	10.56
6	2.60	0.144	100	30.00
7	1.35	0.115	100	14.16
8	2.71	0.106	100	30.00
9	1.97	0.136	100	15.84
10	1.95	0.131	100	17.28

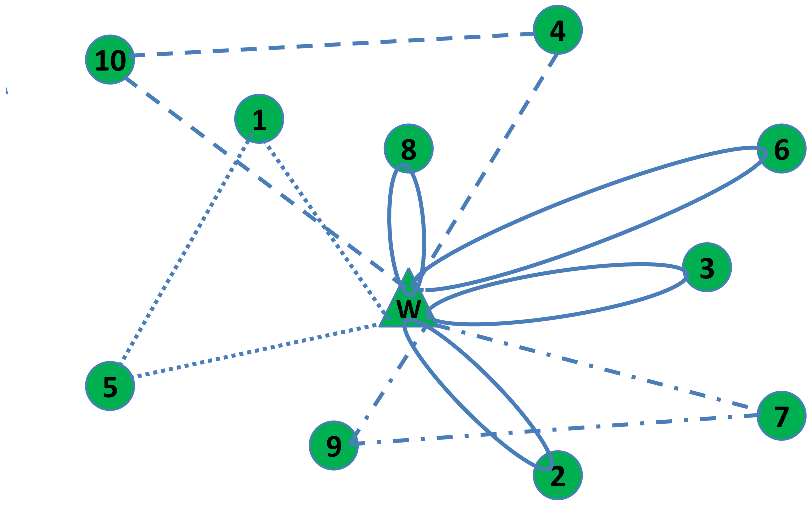

Fig. 4. The optimal solution of the 10 customers for the SP-DIRP.

For instance, route V_{R1} {(5, 1)} delivers 10.56 tons product and 19.44 tons product respectively to customer 5 and customer 1, with a total demand of 30 tons. Based on the delivery quantity to each of customers, we can see that the truck capacity is optimized efficiently. For example, route V_{R5} {(7, 4)} delivers 10.88 tons and 19.12 tons to customer 7 and customer 4, with a total demand of 30 tons, which is similar with the capacity of truck being used for the distribution of the product. Therefore, it shows that the truck loading is optimized efficiently with the average capacity utilization for all tours being 100%.

Furthermore, in the second case, we consider 10 customers which are scattered around the warehouse as shown in Fig. 3, and all the input parameters are the same with the first case. The values of these parameters are then displayed in Table 3.

The generated 7-clients instance of the SP-DIRP is solved by AMPL, with version CPLEX 12.6.3. The optimal solution of the seven customers for SP-DIRP is graphically presented in Fig. 2 and the quantity delivered to the customers is displayed in Table 2. In the solution, only one truck or vehicle with a homogenous capacity is used to replenish the product to each of the customers. As illustrated in Fig. 2, the customers are assigned to five routes {(2), (3), (5, 1), (6), (7, 4)}.

To demonstrate the performance of our proposed model, we solve the medium case problem with 10 customers. In the solution for the second case, we found also only one vehicle with a homogenous capacity is used to replenish the product to each of the customers as illustrated in Fig. 4 above.

For example, route V_{R6} {(9, 7)}, the homogenous vehicle is used to deliver 15.84 tons product and 14.16 tons product respectively from the warehouse to customer 9 and customer 7, with a total demand of 30 tons. Based on the delivery quantity to each of customers, we can see that the vehicle loading capacity is optimized efficiently. Next, for route V_{R7} {(10, 4)}, the vehicle delivers 17.28 tons and 12.72 tons to customer 10 and customer 4, with a total demand of 30 tons, which is similar with the capacity of the vehicle being used for the distribution of the product. As a results, it also shows that the truck loading capacity is optimized efficiently with the average capacity utilization for all tours is 100%.

Hence, the summary results for characteristics of the distribution pattern are shown in Table 4.

Table 4. Summary results for characteristics of the distribution pattern.

Tour	Vehicle load (ton)	Total vehicle load (ton)
V_{R1}= (0,2,0)	30.00	30.00
V_{R2}= (0,3,0)	30.00	30.00
V_{R3}= (0,5,1,0)	10.56 + 19.44	30.00
V_{R4}= (0,6,0)	30.00	30.00
V_{R5}= (0,7,4,0)	10.88 + 19.12	30.00
		150.00
V_{R1}= (0,2,0)	30.00	30.00
V_{R2}= (0,3,0)	30.00	30.00
V_{R3}= (0,5,1,0)	10.56 + 19.44	30.00
V_{R4}= (0,6,0)	30.00	30.00
V_{R5}= (0,8,0)	30.00	30.00
V_{R6}= (0,9,7,0)	15.84 + 14.16	30.00
V_{R7}= (0,10,4,0)	17.28 + 12.72	30.00
		210.00

5. Conclusions

We investigated the single-period deterministic inventory routing problem (SP-DIRP) in which a single warehouse is distributing a single product to a set of customers consuming it at stationary demand rates, using a fleet of homogeneous vehicles over a given finite horizon.

The objective is to determine the optimal quantities to be delivered to the customers, the delivery time, and to design vehicle delivery routes, so that the total transportation and inventory costs are minimized. The SP-DIRP is formulated as a linear mixed-integer program. From the small and medium example case, it shows that the vehicle capacity is optimized efficiently with the average capacity utilization for all tours is 100%.

Further research approach will consist of adapting the existing model and solution to some numerical experiments and to the real life application problems, including a large set of customers. Finally, it is worthwhile to investigate how the approach can be extended from the single-period setting to the multi-period deterministic and stochastic case in the future research.

Acknowledgments

This research was supported by Malaysian Ministry of Higher Education (MOHE) through Research Acculturation Grant Scheme (RAGS), under Grant No. (RAGS/1/2015/SG0/UUM/02/1).

References

1. Rahim, M. K. I. A.; Radzuan, K.; and Yaakob, M. (2016). Integrated analysis of inventory management and transportation systems for the single-period problem. *Advanced Science Letters*, 22(9), 2113-2116.
2. Rahim, M. K. I. A.; Abidin, R.; Iteng, R.; and Lamsali, H. (2016). The integrated model for solving the single-period deterministic inventory routing problem. *AIP Conference Proceedings*, 1761(1), 020006.
3. Rahim, M. K. I. A.; Zhong, Y.; Aghezzaf, E. H. and Aouam, T. (2014). Modelling and solving the multiperiod inventory-routing problem with stochastic stationary demand rates. *International Journal of Production Research*, 52(14), 4351-4363.
4. Dror, M.; Ball, M.; and Golden, B. (1985). A computational comparison of algorithms for the inventory routing problem. *Annals of Operations Research*, 4(1), 2-23.
5. Bertazzi, L.; Paletta, G.; and Speranza, M.G. (2002). Deterministic order-up-to level policies in an inventory routing problem, *Transportation Science*, 36(1), 119-132.
6. Campbell, A.M.; and Savelsbergh, M.W.P. (2004). A decomposition approach for the inventory routing problem. *Transportation Science*, 38(4), 488-502.
7. Abdelmaguid, T. F.; and M. M. Dessouky. (2006). A genetic algorithm approach to the integrated inventory-distribution problem, *International Journal of Production Research*, 44(21), 4445-4464.
8. Archetti, C.; Bertazzi, L.; Laporte, G.; and Speranza, M.G. (2007). A branch-and-cut algorithm for a vendor-managed inventory-routing problem. *Transportation Science*, 41(3), 382-391.
9. Yugang, Y.; Chen, H.X.; and Chu, F. (2008). A new model and hybrid approach for large scale inventory routing problem. *European Journal of Operational Research*, 189(3), 1022-1040.
10. Archetti, C.; Bertazzi, L.; Hertz, A.; and Speranza, M. (2012). A hybrid heuristic for an inventory routing problem. *INFORMS Journal on Computing*, 24(1), 101-116.
11. Gallego, G.; and Simchi-Levi, D. (1990). On the effectiveness of direct shipping strategy for the one warehouse multi-retailer R-systems. *Management Science*, 36(2), 240-243.

12. Anily, S.; and Federgruen, A. (1993). Two-echelon distribution systems with vehicle routing costs and central inventories. *Operations Research*, 41(1), 37-47.
13. Bramel, J.B., and Simchi-Levi, D. (1995). A location based heuristic for general routing problems. *Operations Research*, 43(4), 649-660.
14. Chan, L.M.; Federgruen, A.; and Simchi-Levi, D. (1998). Probabilistic analyses and practical algorithms for inventory-routing models. *Operations Research*, 46(1), 96-106.
15. Aghezzaf E.-H.; Raa B.; and Landeghem H.V. (2006). Modeling inventory routing problems in supply chains of high consumption products. *European Journal of Operational Research*, 169(3), 1048-1063.
16. Jung, J.; and Mathur, K. (2007). An efficient heuristic algorithm for a two-echelon joint inventory and routing problem. *Transportation Science*, 41(1), 55-73.
17. Raa, B.; and Aghezzaf, E.-H. (2008). Designing distribution patterns for long-term inventory routing with constant demand rates. *International Journal of Production Economics*, 112(1), 255-263.