Journal of Engineering Science and Technology
Vol. 12, No. 3 (2017) 701 - 715

DIRECT MATHEMATICAL SOLUTIONS FOR THE GAMMA-RAY DETECTORS GEOMETRICAL AND TOTAL EFFICIENCIES INTEGRABLE FORMULAE

MAHMOUD I. ABBAS, MONA M. GOUDA,
MOHAMED S. BADAWI*, AHMED M. EL-KHATIB

Physics Department, Faculty of Science, Alexandria University, 21511 Alexandria, Egypt
*Corresponding author at: ms241178@hotmail.com

Abstract

A new mathematical solution for the geometrical and total efficiencies integrable equations of cylindrical and spherical gamma-ray detectors is proposed. The efficiencies integrable equations are based on the accurate analytical computation of the photon path length traverses within the detector active medium, and the geometrical solid angle, Ω, subtended by the source to the detector at the point of entrance. The comparison between the efficiencies calculated values by the use of both, the integrable and direct solution formulae, showing very good agreement.

Keywords: Mathematical solution, Gamma-ray detectors, Geometrical and total efficiency.

1. Introduction

The total and the full-energy peak efficiencies for any specific source-to-detector configuration using HPGe or NaI(Tl) detectors is required in the field of gamma-ray spectrometry. Several authors [1-14] have treated the total efficiency and have given useful solutions. Selim and Abbas derived mathematical expressions in the form of elliptic integrable equations, which can be used to calculate the efficiency of any detector shape or type by using any source shape at any geometrical locations [15-18]. These formulae are valid for different detector shapes and give very good agreement, when compared with the corresponding experimental results [19-25].

The work described below involves the mathematical solutions of the integral equations of the total and geometrical efficiencies used by Selim et al. [16] and

Nomenclatures

d_i	Path length through the detector, m
h	Distance between source and the detector, m
L	Detector length, m
R	Detector radius , m
Greek Symbols	
θ	Polar angle, deg
μ	Attenuation coefficient of the detector material, m^{-1}
φ	Azimuthal angle, deg
ε	Detector efficiency with respect to point source
ε_g	Geometrical efficiency

Hamzawy [26] for the cylindrical and spherical detectors, respectively. This study shows the validity of these direct mathematical solution efficiency values and that obtained from the integral efficiency equations.

2. Cylindrical Detector

2.1. The total efficiency

2.1.1. The integrable equation

The total efficiency of a cylindrical detector by using an isotropic radiating axial-point source has been derived by Selim et al. [16] by the use of the spherical coordinate's technique. The geometry of an axial point source to a cylindrical detector, ($2R\times L$), located at distance, h, from the surface is given in Fig. 1. The striking photon may enter the upper face of the detector and emerge from its base or side based on the previous extreme values of the polar angles as shown in Fig. 1., these distances covered by the photon in two cases as a function in the polar angle, θ, and can be given as:

Enter from the face and exit from the base:

$$d_1 = \frac{L}{\cos\theta} \tag{1}$$

Enter from the face and exit from the side:

$$d_2 = \frac{R}{\sin\theta} - \frac{h}{\cos\theta} \tag{2}$$

where θ, is the polar angle as shown in Fig. 1 can be changed from 0 to θ_1, then change from θ_1 till reach to θ_2, as well and given by:

$$\theta_1 = \tan^{-1}\left(\frac{R}{h+L}\right) \qquad \theta_2 = \tan^{-1}\left(\frac{R}{h}\right) \tag{3}$$

This means, there will be different values for, d_1, and d_2, based on the change of the polar angle, which change from 0 to θ_1, then change from θ_1, till reach to θ_2, respectively.

Now after the above discussion and made the integration over the azimuthal angle, φ, which takes always the values (from 0 to 2π), the total efficiency will be function only in polar angle, θ, and is given by [16]:

$$\varepsilon = \frac{1}{4\pi}\int_{\theta}\int_{\varphi}(1 - e^{-\mu.d_i})\ \sin\theta\, d\varphi\, d\theta = \frac{1}{2}\left(\int_{0}^{\theta_1} f_1\, d\theta + \int_{\theta_1}^{\theta_2} f_2\, d\theta\right) \tag{4}$$

where,

$$f_i = (1 - e^{-\mu.d_i}).\sin\theta\ ,\quad with\ i = 1,2 \tag{5}$$

While, d_i, are the possible path lengths travelled by the photon within the detector active volume, d_1 and d_2, as discussed before. At the same time as, μ, is the total attenuation coefficient of the detector material at the γ-ray energy, E_γ, where the coherent scattering part was excluded.

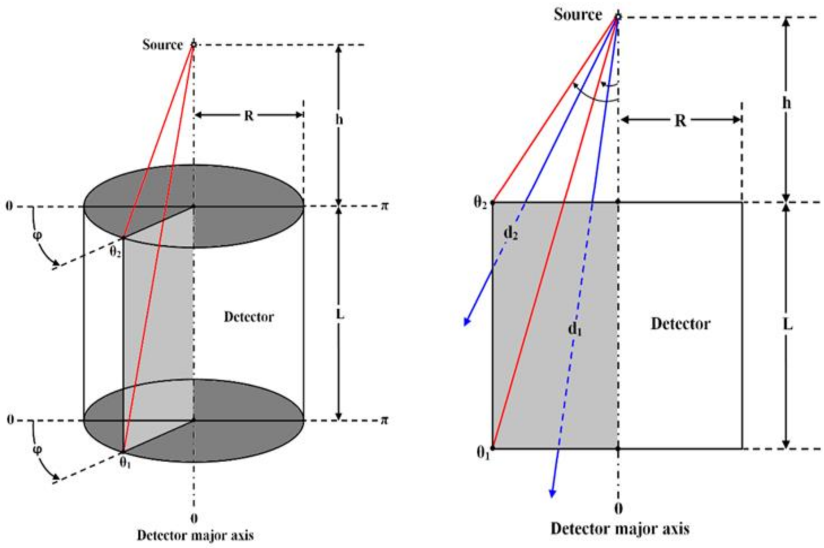

Fig. 1. Axial point source with cylindrical detector.

2.1.2. The solution of the integral equation

The total efficiency equation Eq. (4) has two integrable parts. Each integral is very complicated and cannot be solved it by using the normal integral rules directly, but can be solved by using the Maclaurin's series, so they obtained solution can be considered as series sum, then each term of these series solved alone and then the final approximate solution of the integral equation has to be in the next form:

$$\varepsilon = \frac{1}{2}(Y_1 + Y_2) \tag{6}$$

where,

$$Y_1 = \int_0^{\theta_1} \left(1 - e^{-\mu.L.\sec\theta}\right).\sin\theta d\theta$$

Let $a = -\mu.L$, so that,

$$Y_1 = \int_0^{\theta_1} \left(1 - e^{a.\sec\theta}\right).\sin\theta d\theta$$

then,

$$Y_1 = a.\ln\left|\cos\theta_1\right| - \sum_{n=1}^{\infty} \frac{a^{n+1}}{n(n+1)!}\left(\sec^n\theta_1 - 1\right) \tag{7}$$

and,

$$Y_2 = \int_{\theta_1}^{\theta_2} \left(1 - e^{-\mu\left(\frac{R}{\sin\theta} - \frac{h}{\cos\theta}\right)}\right).\sin\theta d\theta$$

Let

$x = \mu.h$, $y = \mu.R$,

so that,

$$Y_2 = \int_{\theta_1}^{\theta_2} \left(1 - e^{x.\sec\theta - y.\csc\theta}\right).\sin\theta.d\theta$$

then,

$$\begin{aligned} Y_2 = {} & y\left(\theta_2 - \theta_1\right) + x\left(\ln\left|\frac{\cos\theta_2}{\cos\theta_1}\right| + y.\mathrm{P}_1\right) - \frac{y^2}{2}\left(V_0 + x.V_1\right) \\ & - \sum_{n=2}^{\infty}\frac{x^n}{n!}\left(\frac{\sec^{n-1}\theta_2 - \sec^{n-1}\theta_1}{n-1} - y.\mathrm{P}_n + \frac{y^2}{2}\mathrm{V}_n\right) \\ & + \sum_{m=2}^{\infty}\frac{(-1)^m.y^{m+1}}{(m+1)!}\left(\mathrm{X}_m + x.\mathrm{A}_m\right) \\ & + \sum_{m=2}^{\infty}\frac{(-1)^m.y^{m+1}}{(m+1)!}\sum_{n=2}^{\infty}\frac{x^n}{n!}J_{n,m} \end{aligned} \tag{8}$$

where,

$$\mathrm{P}_n = \frac{\sec^{n-2}\theta_2.\tan\theta_2 - \sec^{n-2}\theta_1.\tan\theta_1}{n-1} + \frac{n-2}{n-1}\mathrm{P}_{n-2} \tag{9}$$

$$V_n = \frac{\sec^{n-1}\theta_2 - \sec^{n-1}\theta_1}{n-1} + V_{n-2} \quad (10)$$

$$X_m = \frac{\csc^{m-2}\theta_1.\cot\theta_1 - \csc^{m-2}\theta_2.\cot\theta_2}{m-1} + \frac{m-2}{m-1}X_{m-2} \quad (11)$$

$$A_m = \frac{\csc^{m-1}\theta_1 - \csc^{m-1}\theta_2}{m-1} + A_{m-2} \quad (12)$$

$$J_{n,m} = \sec^{n-1}\theta_1.\csc^{m-1}\theta_1 - \sec^{n-1}\theta_2.\csc^{m-1}\theta_2 + n.J_{n,m-2} - (m-2).J_{n-2,m} \quad (13)$$

$$J_{n,0} = P_n \quad (14)$$

$$J_{n,1} = V_n \quad (15)$$

$$J_{0,m} = X_m \quad (16)$$

$$J_{1,m} = A_m \quad (17)$$

$$P_1 = A_0 = \ln\left|\frac{\sec\theta_2 + \tan\theta_2}{\sec\theta_1 + \tan\theta_1}\right| \quad (18)$$

$$X_1 = V_0 = \ln\left|\frac{\tan\left(\theta_2/2\right)}{\tan\left(\theta_1/2\right)}\right| \quad (19)$$

$$V_1 = A_1 = \ln\left|\frac{\tan\theta_2}{\tan\theta_1}\right| \quad (20)$$

2.2. The geometrical efficiency

The geometrical efficiency of a cylindrical detector by using an isotropic radiating axial-point source is obtained by putting, $\mu=\infty$, in Eq. (4). Hence, the geometrical efficiency is expressed as:

$$\varepsilon_g = \frac{1}{2}\left(\int_0^{\theta_1}\sin\theta.d\theta + \int_{\theta_1}^{\theta_2}\sin\theta.d\theta\right) \quad (21)$$

$$\varepsilon_g = 1 - \cos\theta_2 \quad (22)$$

3. Spherical Detector

3.1. The total efficiency

3.1.1. The integrable equation

Figure 2 illustrates a spherical detector with radius, R. There is only one allowed path length, which is given by:

$$d = 2\sqrt{R^2 - (R+h)^2\sin^2\theta} \quad (23)$$

From the spherical detector symmetry, the azimuthal angle, φ, takes only the value (2π) and the polar angle, θ, has one step only:

$$\theta_{max} = \sin^{-1}\left(\frac{R}{R+h}\right) \tag{24}$$

Then, the efficiency is given by:

$$\varepsilon = \frac{1}{2}\left(\int_0^{\theta_{max}} \left(1 - e^{-\mu.d}\right) \sin\theta.d\theta\right) \tag{25}$$

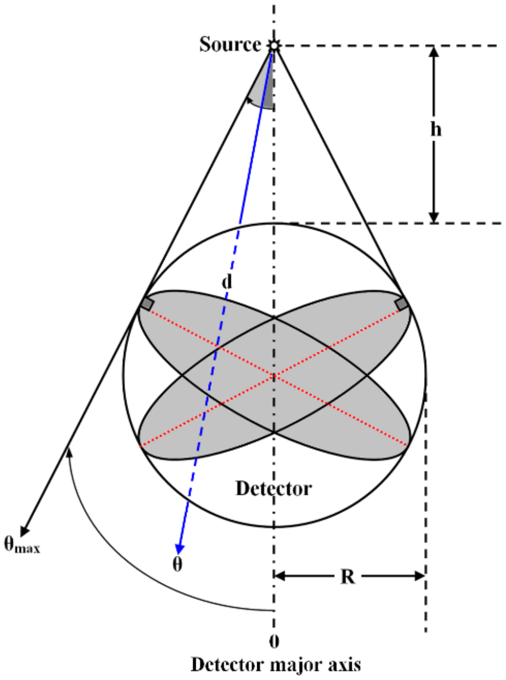

Fig. 2. Schematic Diagram of the spherical detector for an isotropic radiating axial point source.

3.1.2. Solution of the integrable equation

The solution of Eq. (25) is given as follows:

$$\varepsilon = \frac{1}{2}\left(\int_0^{\theta_{max}} \left(1 - e^{-2\mu.\sqrt{R^2-(R+h)^2.\sin^2\theta}}\right) = \frac{1}{2}\int_0^{\theta_{max}} \sin\theta.d\theta - \frac{1}{2}\text{II}.\sin\theta.d\theta\right)$$

$$\varepsilon = \frac{1}{2}(1 - \cos\theta_{max}) - \frac{1}{2}\text{II}$$

where, $\text{II} = \int_0^{\theta_{max}} e^{-2.\mu.\sqrt{R^2-(R+h)^2.\sin^2\theta}}.\sin\theta.d\theta$

Let, $a = R + h$, $b = -2\mu$, so that,

$$\text{II} = \int_0^{\theta_{max}} e^{b.\sqrt{R^2-a^2.\sin^2\theta}}.\sin\theta.d\theta \tag{26}$$

Let $x = \sqrt{R^2 - a^2\sin^2\theta}$, therefore,

$$dx = \frac{1}{2}\left(R^2 - a^2\sin^2\theta\right)^{-\frac{1}{2}}.\left(-2.a^2.\sin\theta.\cos\theta\right)d\theta$$

$$\therefore dx = \frac{-a^2.\sin\theta.\cos\theta}{\sqrt{R^2 - a^2.\sin^2\theta}}.d\theta$$

The new limits of the integral can be determined as follow:

At $\theta_{\max} = \sin^{-1}\frac{R}{R+h} \quad \rightarrow \quad x = 0$

At $\theta = 0 \quad \rightarrow \quad x = R$

where, $x = \sqrt{R^2 - a^2\sin^2\theta}$

then, $\sin\theta = \frac{\sqrt{R^2 - x^2}}{a}$, so, $\cos\theta = \frac{\sqrt{a^2 - R^2 + x^2}}{a}$

By substituting of dθ and cosθ, in Eq.(26):

$$\text{II} = -\int_R^0 \frac{x.e^{b.x}}{a.\sqrt{a^2 - R^2 + x^2}}.dx = \frac{1}{a}\int_0^R \frac{x.e^{b.x}}{\sqrt{a^2 - R^2 + x^2}}.dx$$

Let $f = a^2 - R^2$, so that,

$$\text{II} = \frac{1}{a}\int_0^R \frac{x.e^{b.x}}{\sqrt{f + x^2}}.dx$$

Then, by applying the trapezoidal rule on the previous integral, the solution is:

$$\text{II} = \frac{k}{2.a}\left(\frac{e^{b.R}.R}{\sqrt{f + R^2}} + 2.\sum_{m=1}^{n-1}\frac{e^{b.m.k}.(m.k)}{\sqrt{f + (m.k)^2}}\right)$$

where $k = \frac{R}{n}$, then,

$$\varepsilon = \frac{1}{2}(1 - \cos\theta_{\max}) - \frac{k.R}{4.a^2}e^{b.R} + \frac{k}{2.a}\sum_{m=1}^{n-1}\frac{e^{b.m.k}.(m.R)}{\sqrt{a^2.n^2 + R^2.(m^2 - n^2)}} \qquad (27)$$

3.2. The geometrical efficiency

The geometrical efficiency of the spherical detector is obtained by putting, $\mu=\infty$, in Eq. (25). Hence, the geometrical efficiency is expressed as:

$$\varepsilon_g = \frac{1}{2}\int_0^{\theta_{\max}}\sin\theta.d\theta = \frac{1}{2}\left(1 - \cos\theta_{\max}\right)$$

then,

$$\varepsilon_g = \frac{1}{2}\left(1 - \frac{\sqrt{(R+h)^2 - R^2}}{R}\right) \qquad (28)$$

4. Validation of the Present Method

4.1. Cylindrical detector

The total efficiency of a cylindrical (2.54×2.54 cm^2) Ge detector (in the energy range 2 - 16 MeV) using an isotropic radiating axial point source placed at different heights, *h* = 0, 0.5, 1, 1.5 and 2 cm, above the surface of the detector has been calculated for the two methods (the integral method and its direct mathematical solution method) by using basic program. The comparison between the total efficiencies in the two methods is illustrated in Figs. 3 - 7.

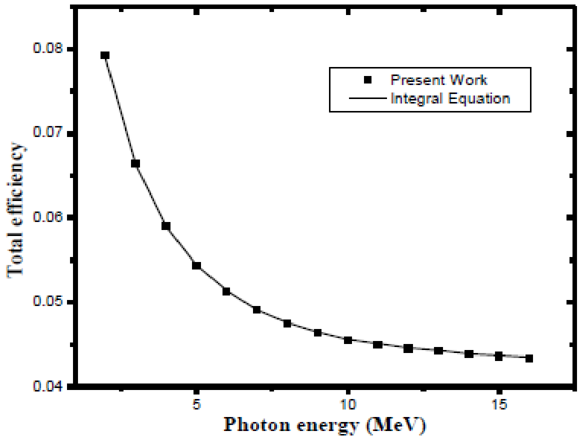

Fig. 3. Variation of total efficiency with the photon energy for an isotropic radiating axial point source placed in contact with the surface of 1″×1″ cylindrical Ge detector.

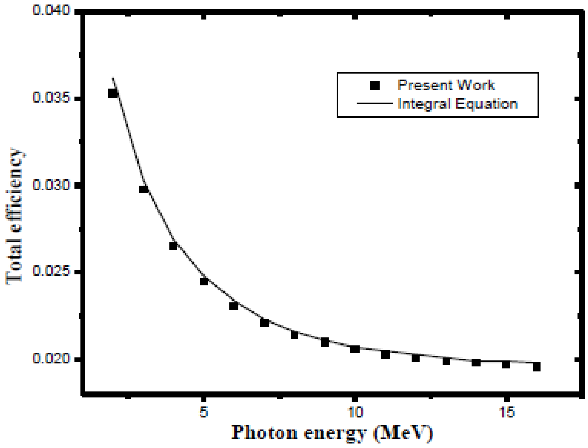

Fig. 4. Variation of total efficiency with the photon energy for an isotropic radiating axial point source placed at 0.5 cm from the surface of 1″×1″ cylindrical Ge detector.

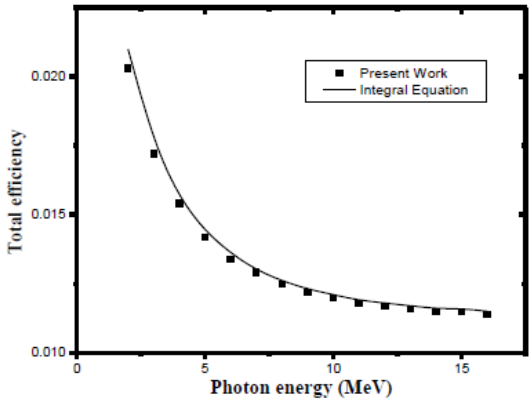

Fig. 5. Variation of total efficiency with the photon energy for an isotropic radiating axial point source placed at 1 cm from the surface of 1″×1″ cylindrical Ge detector.

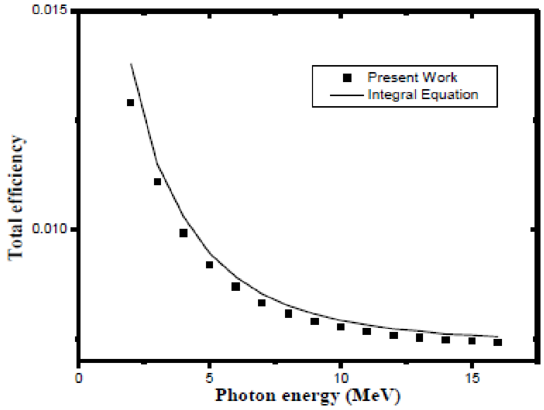

Fig. 6. Variation of total efficiency with the photon energy for an isotropic radiating axial point source placed at 1.5 cm from the surface of 1″×1″ cylindrical Ge detector.

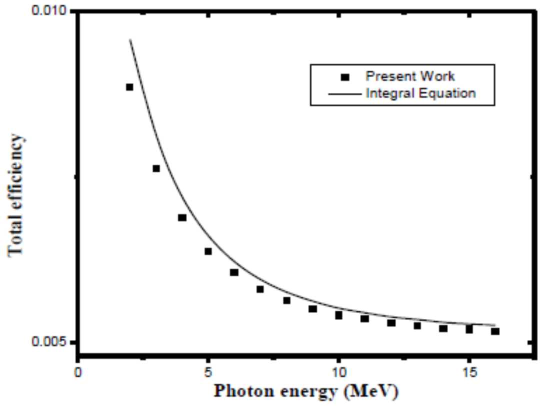

Fig. 7. Variation of total efficiency with the photon energy for an isotropic radiating axial point source placed at 2 cm from the surface of 1″×1″ cylindrical Ge detector.

4.2. Spherical detector

The total efficiency of spherical (with radius = 2 cm) Ge and NaI detectors (in the energy range 0.1 - 5 MeV) using an isotropic radiating axial point source placed at different heights, *h*=0.5, 5 and 10 cm above the surface of the detectors has been calculated. The efficiency calculations have been performed by using the two methods (the integral equation method and its direct mathematical solution method). The comparisons between the total efficiencies in the two methods are illustrated in Figs. 8 - 13.

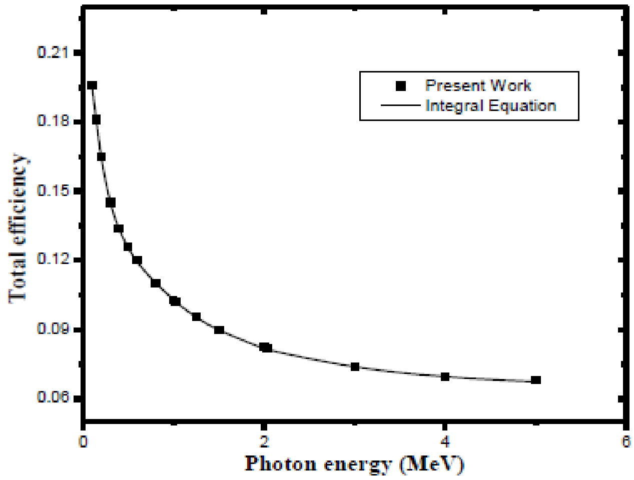

Fig. 8. Variation of total efficiency with the photon energy for an isotropic radiating axial point source placed at 0.5 cm from the surface of a spherical Ge detector with radius 2 cm.

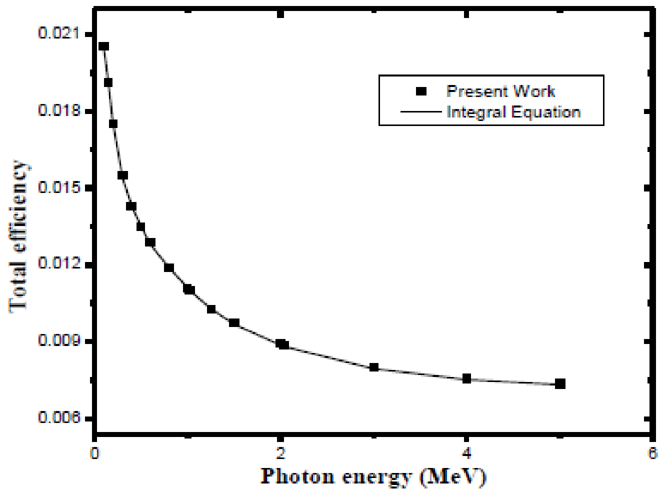

Fig. 9. Variation of total efficiency with the photon energy for an isotropic radiating axial point source placed at 5 cm from the surface of a spherical Ge detector with radius 2 cm.

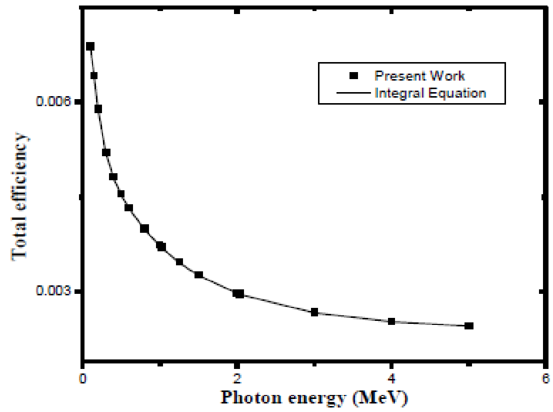

Fig. 10. Variation of total efficiency with the photon energy for an isotropic radiating axial point source placed at 10 cm from the surface of a spherical Ge detector with radius 2 cm.

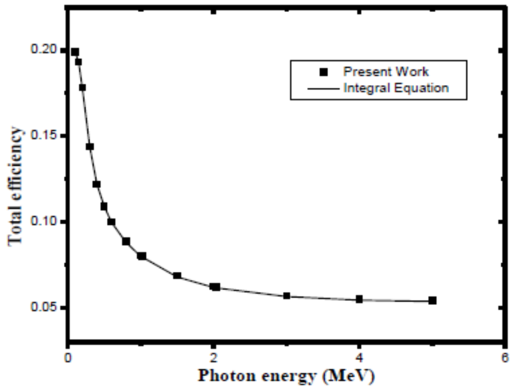

Fig. 11. Variation of total efficiency with the photon energy for an isotropic radiating axial point source placed at 0.5 cm from the surface of a spherical NaI(Tl) detector with radius 2 cm.

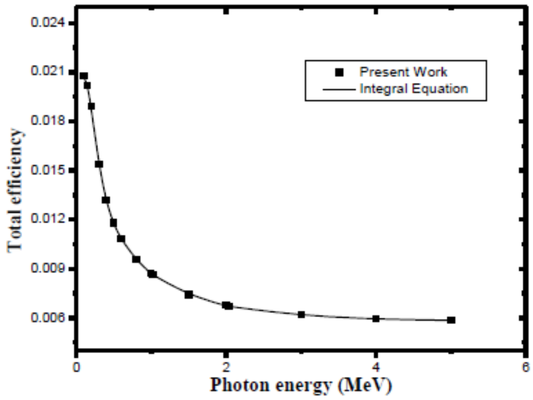

Fig. 12. Variation of total efficiency with the photon energy for an isotropic radiating axial point source placed at 5 cm from the surface of a spherical NaI(Tl) detector with radius 2 cm.

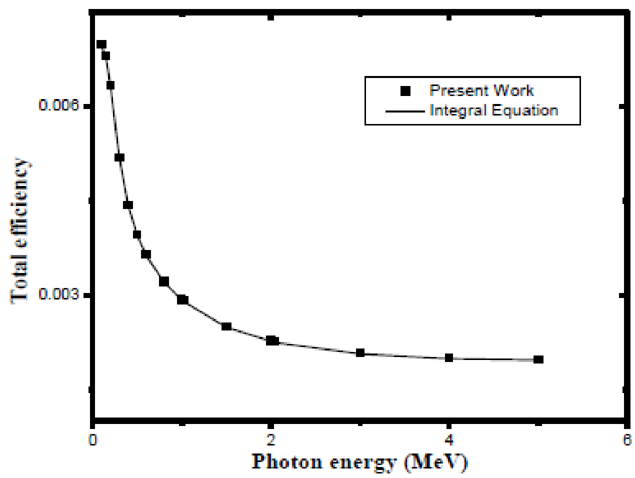

Fig. 13. Variation of total efficiency with the photon energy for an isotropic radiating axial point source placed at 10 cm from the surface of a spherical NaI(Tl) detector with radius 2 cm.

In addition, the intrinsic efficiency of a spherical (with radius = 4.37 cm) NaI(Tl) detector (in the energy range from 0.105 up to 7.9 MeV) by the use of an isotropic radiating axial point source placed at two heights, h=2.185 and 8.74 cm above the surface of the detector has been calculated. The calculated efficiency values (present work) are compared with the published values [27], as shown in Figs. 14 and 15.

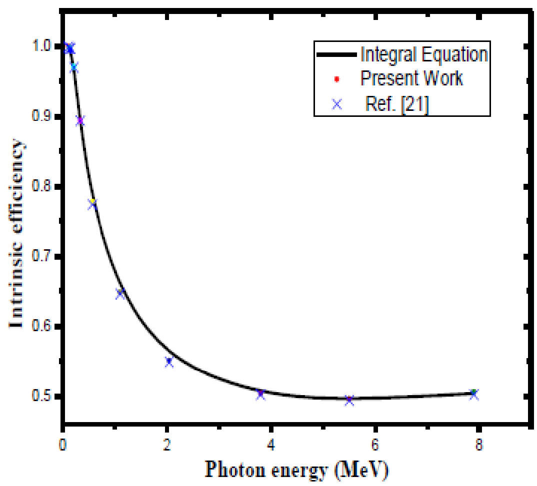

Fig. 14. Variation of intrinsic efficiency with the photon energy for an isotropic radiating axial point source placed at 2.185 cm from the surface of a spherical NaI(Tl) detector with radius 4.37cm.

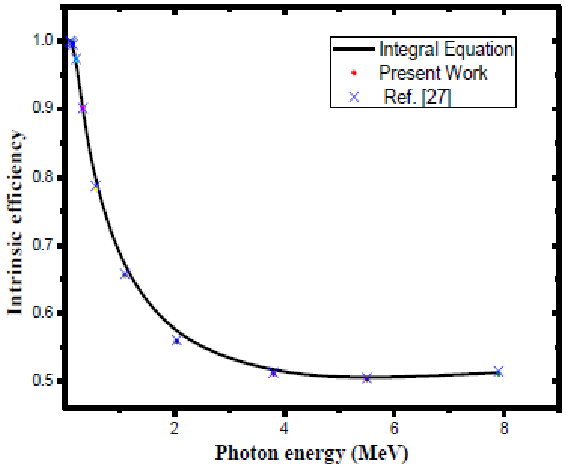

Fig. 15. Variation of intrinsic efficiency with the photon energy for an isotropic radiating axial point source placed at 8.74 cm from the surface of a spherical NaI(Tl) detector with radius 4.37 cm.

5. Conclusions

In this study, the calculation of the total and geometrical efficiencies for cylindrical and spherical [HPGe and NaI(Tl)] detectors with different dimensions have been done by using the following methods: first, is the direct method, which used Simpson or trapezoidal rules to solve the efficiency integrable equations and to compute them numerically by computer with high accuracy. Second, is the solution of the integrable equations (present work). There is a very good agreement between the two methods for all energies and the published experimental and theoretical efficiency values for both the spherical and cylindrical detectors.

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