Fault Detection and Diagnosis Using Cubature Kalman Filter for Nonlinear Process Systems

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***Abstract*—This paper presents a fault detection and diagnosis (FDD) for a nonlinear systems using multiple Cubature Kalman Filter (CKF) model. The proposed scheme able to identify sensors and actuators fault even with the presences of process and measurement noise. Comparison between actual faults with expected fault trajectory enables the FDD to narrow down possible scenario. The utilization of continuous stirred tank reactor (CSTR) simulation illustrates the performance of the scheme in nonlinear system. Result of the study shows the proposed method works effectively in determine the type of fault occurs in the CSTR.**

***Index Terms*—Cubature Kalman Filter; Bank of Residual; CSTR.**

I. INTRODUCTION

Manufacturing plants can have many sensors and actuators all working together to ensure the process works perfectly. But even with state of the art components, manufacturing plants can still encounter degradation, which later can cause malfunctions [1].

Fault is an unpermitted deviation of at least one characteristic property or parameter of the system from the acceptable/usual/standard condition [2]. Researchers currently opt to analytical redundancy in developing fault detection and diagnosis (FDD) due to its ability to reduce the numbers of hardware used in the systems [3]. Model based method is one such example, it involves the process model, which is obtained by identification of the system. Based on the model, the consistency of the measured actual output and estimated output is monitored.

Kalman Filter has been known to provide a very good state estimation by using statistical representation of the system. It becomes the basic structure for recent estimator. Such as Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) which their ability to handle nonlinearity is proven [4,5]. The recent extension to Kalman filter is Cubature Kalman Filter (CKF), which based on third degree spherical radial cubature rule [6]. It achieves success since its introduction [7-9], and recently has come to the attention of FDD [5, 10].

FDD such as [11,12] focus much on sensor fault and other generals disturbance. F. Pierri developed an observer based FDD for sensor faults [1]. Unknown input observer (UIO) has been J. Zarei main focus in designing FDD, and it involved sensor fault [13]. But sensors fault is not always the main contribution to system failure, actuator fault can also produce the same effect to the system. The numbers of research focusing on isolating sensor and actuator faults has seen a rise in FDD area, this to show the importance of identifying faults.

Motivated by this consideration, this research focus on FDD for sensor and actuator fault using CKF. The proposed method FDD consist of multiple CKF model to generate a set of residuals, this residual is later compared with another set of expected fault trajectory to identify type of faults. The main contribution of this work is the exploitation CKF ability to estimate and diagnose sensor and actuator faults.

This paper is organized as follows. Section II provides the formulation for nonlinear system with actuator and sensors faults. It includes reviewing the general concept of CKF discussion the proposed FDD structure to detect and identify faults. Setup for scheme simulation is describe in section III. Simulation results obtained from a non-linear Continuous Stirred Tank Reactor (CSTR) are presented in Section IV, and conclusions are presented in Section V.

II. METHODOLOGY

A. Problem Formulation

Consider a nonlinear discrete time system with sensors and actuator faults as:

$$\begin{aligned} x_k &= f(x_{k-1}, u_{k-1} + \tilde{u}) + v_{k-1} \\ y_k &= h(x_k, u_k) + \tilde{y} + w_k \end{aligned} \tag{1}$$

where x_k is the state of the dynamic system at discrete time k, $f(.)$ and $h(.)$ are some known functions, u_k is the known control input, y_k is the measurement, $\tilde{u}$ denotes the unknown fault vector for the actuators, $\tilde{y}$ denotes the unknown fault vector for the sensors, v_{k-1} and w_k are independent process and measurement Gaussian noise sequences with zero means and covariance's Q_{k-1} and R_k, respectively. On the receipt of a new measurement at time k, we update the old posterior density of the state at time $k-1$.

In this work, we consider the problem of FDD for at most three faults. It encompasses the cases combination of two actuators and a single sensor. Since a large number of simultaneous faults would occur less frequently, the consideration of proposed faults would meet most of the practical needs. Preparatory to the presentation of the FDD mechanism, we review the CKF to estimate the nonlinear model system states.

B. Cubature Kalman Filter (CKF) Algorithm

A set of model estimators is required to develop. The number of estimator depends on number of expected faulty state trajectory. Each estimator requires to be filtered using CKF. The following is the steps required in implementing the scheme:

a. Time Update

1. Assume at time k that the posterior density function

   $p(x_{k-1} | D_{k-1}) = N(\hat{x}_{k-1|k-1}, P_{k-1|k-1})$ is known.

   Factorize

$$P_{k-1|k-1} = S_{k-1|k-1} S^T_{k-1|k-1} \tag{2}$$

2. Evaluate the cubature points $(i = 1,2,...,m)$

$$X_{i,k-1|k-1} = S_{k-1|k-1} x_i + \hat{x}_{k-1|k-1} \tag{3}$$

   where $m = 2n_x$.

3. Evaluate the propagated cubature points $(i = 1,2,...,m)$

$$X^*_{i,k|k-1} = f(X_{i,k-1|k-1}, u_{k-1}) \tag{4}$$

4. Estimate the predicted state

$$\hat{x}_{k|k-1} = \frac{1}{m} \sum_{i=1}^{m} X^*_{i,k|k-1} \tag{5}$$

5. Estimate the predicted error covariance

$$P_{k|k-1} = \frac{1}{m} \sum_{i=1}^{m} X^*_{i,k|k-1} X^{*T}_{i,k|k-1} - \hat{x}_{k|k-1} \hat{x}^T_{k|k-1} + Q_{k-1} \tag{6}$$

b. Measurement Update

1. Factorize

$$P_{k|k-1} = S_{k|k-1} S^T_{k|k-1} \tag{7}$$

2. Evaluate the cubature points $(i = 1,2,...,m)$

$$X_{i,k|k-1} = S_{k|k-1} x_i + \hat{x}_{k|k-1} \tag{8}$$

3. Evaluate the propagated cubature points $(i = 1,2,...,m)$

$$Y_{i,k|k-1} = h(X_{i,k|k-1}, u_k) \tag{9}$$

4. Estimate the predicted measurement

$$\hat{y}_{k|k-1} = \frac{1}{m} \sum_{i=1}^{m} Y_{i,k|k-1} \tag{10}$$

5. Estimate the innovation covariance matrix

$$P_{zz,k|k-1} = \frac{1}{m} \sum_{i=1}^{m} Y_{i,k|k-1} Y^T_{i,k|k-1} - \hat{y}_{k|k-1} \hat{y}^T_{k|k-1} + R_k \tag{11}$$

6. Estimate the cross-covariance matrix

$$P_{xz,k|k-1} = \frac{1}{m} \sum_{i=1}^{m} X_{i,k|k-1} Y^T_{i,k|k-1} - \hat{x}_{k|k-1} \hat{y}^T_{k|k-1} \tag{12}$$

7. Estimate the Kalman gain

$$W_k = P_{xz,k|k-1} P^{-1}_{zz,k|k-1} \tag{13}$$

8. Estimate the updated state

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + W_k (y_k - \hat{y}_{k|k-1}) \tag{14}$$

9. Estimate the corresponding error covariance

$$P_{k|k} = P_{k|k-1} - W_k P_{zz,k|k-1} W^T_k \tag{15}$$

C. Fault Detection

In practical applications, it is necessary that generated residuals be robust against disturbance, noise and uncertainties [13]. If the designed state estimation is stable, the state estimation error converges to zero asymptotically. Residual is commonly used to detect fault occurring in the system [3,14]. Thus, in the steady state under fault free condition, residual formulation:

$$r_k = y_k - \hat{y}_k = 0 \tag{16}$$

where $\hat{y}_k$ is the estimated measurement. This signal should deviate from zero (zero mean) when a fault occurs i.e. $r_k \neq 0$.

D. Fault Diagnosis

Fault diagnosis is a task consists of determining of the fault type as details as possible [18]. To identify fault, comparison of the measured signal with the expected trajectory of the process need to be done. This common technique for fault identification purpose is to generate a symptom signal, which is called error. Error signal has similar characteristics as a residual, but is generated by comparing estimated measurement with possible expected trajectory.

Comparing the faulty scenario with expected trajectory requires a certain index of similarities and contrast. The higher the error signal shows large differences between the two signals while low error signals shows similarity of the compared signals.

$$e_{sc,hyp} = \hat{y}_{sc} - \bar{y}_{hyp} \tag{17}$$

where $e_{sc,hyp}$ denotes the generated error between estimated measurement and expected trajectory, $\hat{y}_{sc}$ denotes estimated measurement, $\bar{y}_{hyp}$ denote the estimated expected trajectory.

Expected trajectory is the hypothesize process output with one or more faults introduced to the process models. For each faulty scenario, the expected process trajectory is computed using the process model and the state estimates generated by the CKF that is subjected to sensors and actuators faults. The number of expected faulty state trajectory depends on the number of sensors and actuators.

$$q_f = 2^m \tag{18}$$

where q_f denotes numbers of expected faulty state trajectory, m denotes numbers of sensors/actuators.

For the case of 2 actuators and 1 sensor, the following fault combination is presented. $\theta_i =$, $i = 0...,7$,can be defined as follows; $\theta_0 =$ no fault, $\theta_1 = (\tilde{u}_1)$, $\theta_2 = (\tilde{y}_1)$, $\theta_3 = (\tilde{u}_1, \tilde{y}_1)$, $\theta_4 = (\tilde{y}_2)$, $\theta_5 = (\tilde{u}_1, \tilde{y}_2)$, $\theta_6 = (\tilde{y}_1, \tilde{y}_2)$, $\theta_7 = (\tilde{u}_1, \tilde{y}_1, \tilde{y}_2)$.

This method is a modification from a much complex concept from Miao Du [16] and Bo Ding [11]. Each of the fault combinations will be used to generate fault trajectory, depends on the number of fault scenario. The produced expected trajectory is $\bar{y}_0$, $\bar{y}_1$, $\bar{y}_2$, $\bar{y}_3$, $\bar{y}_4$, $\bar{y}_5$, $\bar{y}_6$, $\bar{y}_7$.

These expected fault trajectory is compared with actual measurement of the process to generate error. The generated error is as follows: $e_{sc,0} = \hat{y}_{sc} - \bar{y}_0$, $e_{sc,1} = \hat{y}_{sc} - \bar{y}_1$, $e_{sc,2} = \hat{y}_{sc} - \bar{y}_2$, $e_{sc,3} = \hat{y}_{sc} - \bar{y}_3$, $e_{sc,4} = \hat{y}_{sc} - \bar{y}_4$, $e_{sc,5} = \hat{y}_{sc} - \bar{y}_5$, $e_{sc,6} = \hat{y}_{sc} - \bar{y}_6$, $e_{sc,7} = \hat{y}_{sc} - \bar{y}_7$, where sc is the scenario of the current estimated measurement.

The generated error will be monitored to determine the type of fault occurred. If these are done correctly, the error with the faulty scenario will always be zero and the type of fault occurred can be determined. Error in scenario 5 with expected trajectory 5 will be equal to zero. This is due to expected trajectory 5 signal is the same with scenario 5. Other error will produce a certain amount of value depends on the similarity of scenario 5 with other expected trajectory. Set of errors generated:
$e_{5,0} \neq 0$, $e_{5,1} \neq 0$, $e_{5,2} \neq 0$, $e_{5,3} \neq 0$, $e_{5,4} \neq 0$, $e_{5,5} = 0$, $e_{5,6} \neq 0$, $e_{5,7} \neq 0$.

Due to the nonlinearity, complex system, and actuator faults, the signal produce can have a very complex nature. Constant changes and drift in the system is produced, which make the identification process much more complicated. Thus, errors which have scenario similar to expected trajectory will not always be zero, but a smaller value when it is put side by side to other error, shows is the close similarity. $e_{5,5} \approx small$

Threshold can be used to separate the small error with large error. But for this research, we proposed root mean squared error (RMSE) to generate better result. RMSE usually used for determine the performance of certain estimation by enhance the error produced to clearly see the characteristic of the signal. For this case it is used to evaluate the close similarity between the measured signal and expected fault trajectory. Zero is a good indication of very high similarity. $RMSE(e_{5,5}) \approx 0$

III. Simulation

A. Continuous Stirred Tank Reactor (CSTR)

In this section a nonlinear chemical process adopted from [13] is considered to illustrate the efficiency of the proposed method. Figure 1 show the schematic of CSTR.

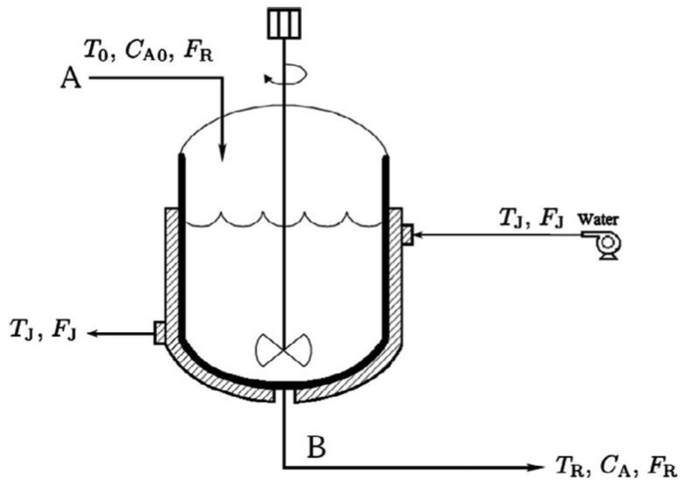

Figure 1: CSTR schematic

The CSTR model is described by the following equation.

$$\begin{aligned}
\frac{dC_A}{dt} &= \frac{F_R}{V_R}(C_{A0} - C_A) - k_0 e^{-E_a/RT_R} C_A \\
\frac{dT_R}{dt} &= \frac{F_R}{V_R}(T_0 - T_R) + \frac{(hA_T)_i}{V_R(\rho C_p)_f}(T_R - T_W) + \frac{-(\Delta H_R)}{(\rho C_p)_f} k_0 e^{-E_a/RT_R} C_A \\
\frac{dT_W}{dt} &= \frac{(hA_T)_i}{V_W(\rho C_p)W}(T_R - T_W) - \frac{(hA_T)_e}{V_W(\rho C_p)W}(T_W - T_J) \\
\frac{dT_J}{dt} &= \frac{F_J}{V_J}(T_{J0} - T_J) + \frac{(hA_T)_e}{V_J(\rho C_p)_J}(T_W - T_J)
\end{aligned} \tag{19}$$

The model is rewritten in normalized dimensionless form as:

$$\begin{aligned}
\frac{dx_1}{dt} &= p_1 u_1 + p_1 u_1 u_2 - p_1 u_2 x_1 - p_2 e^{-p_3/(1+x_2)}(1 + x_1) - p_1 x_1 \\
\frac{dx_2}{dt} &= p_1 u_3 + p_1 u_2 u_3 - p_1 x_2 - p_1 u_2 x_2 - p_4 x_2 \\
&\quad + p_4 x_3 + p_5 p_2 e^{-p_3/(1+x_2)}(1 + x_1) \\
\frac{dx_3}{dt} &= p_6 x_2 - p_6 x_3 - p_7 x_3 + p_7 x_4 \\
\frac{dx_4}{dt} &= p_8 u_4 + p_8 u_4 u_5 - p_8 x_4 - p_8 u_5 x_4 + p_9 x_3 - p_9 x_4
\end{aligned} \tag{20}$$

Table 1
Normalized CSTR Model Parameters

Parameter	Expression	Value	Unit
p_1	$\frac{F_R}{V_R}$	3.333×10^{-2}	s^{-1}
p_2	k_0	4.08×10^{7}	s^{-1}
p_3	$\frac{E_a}{RT_R}$	25.347	–
p_4	$\frac{(hA_T)i}{V_R(\rho C_p)_f}$	6.63×10^{-1}	s^{-1}
p_5	$\frac{-(\Delta H_R)C_A}{(\rho C_p)_f T_R}$	1.45	–
p_6	$\frac{(hA_T)_i}{V_W(\rho C_p)W}$	5.97	s^{-1}
p_7	$\frac{(hA_T)_e}{V_W(\rho C_p)W}$	5.97	s^{-1}
p_8	$\frac{F_J}{V_J}$	1.67×10^{-1}	s^{-1}
p_9	$\frac{(hA_T)_e}{V_J(\rho C_p)_J}$	1.33	s^{-1}

where p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8 and p_9 are summarized in table 1 above and the control input $u_1 = u_2 = u_3 = u_4 = u_5 = 1$.

The parameters of the designed filter are specified as:

$Q = \text{diag}\{10^{-6}, 10^{-6}, 10^{-6}, 10^{-6}\}$,

$R = \text{diag}\{10^{-6}, 10^{-6}, 10^{-6}\}$,

$x_0 = [-0.9754, 0.4725, 0.4269, 0.3793]^T$,

where Q, R, x_0 are applied to generate simulation data. Details of the given simulation setup can be referred in [5,13]

B. Fault Scenario Simulation

In order to determine the effectiveness of the proposed fault detection mechanism, multiple scenarios have been carried out. A scenario is a possible fault combination that the actual system can encounter. Even though, the CSTR simulation contain 5 actuators and 4 outputs, we only take into consideration for scenario which deals with fault $\tilde{u}_1$, $\tilde{y}_1$, and $\tilde{y}_2$ only. Thus the number and combination of fault scenario is similar to the number of expected trajectory.

The total simulation time is k=100, and the sensor and actuator faults are described as follows:

$$\tilde{u}_1 = \begin{cases} 0, & 0 \le k < 25 \\ 0.5, & 25 \le k \le 100 \end{cases} \quad (21)$$

$$\tilde{y}_1 = \{ \; 0, \quad 0 \le k \le 100 \quad (22)$$

$$\tilde{y}_2 = \begin{cases} 0, & 0 \le k < 25 \\ 0.5, & 25 \le k \le 100 \end{cases} \quad (23)$$

IV. RESULT

Figure 2 shows the comparison between estimated CKF output and the measured output. This is to show the ability of the CKF to produce good estimation of the measured signal. Convergence of the estimation can be seen in the early steps of the CKF output. Figure 3 shows CSTR output within scenario 5 (fault type 5), fault occurred at time k=25.

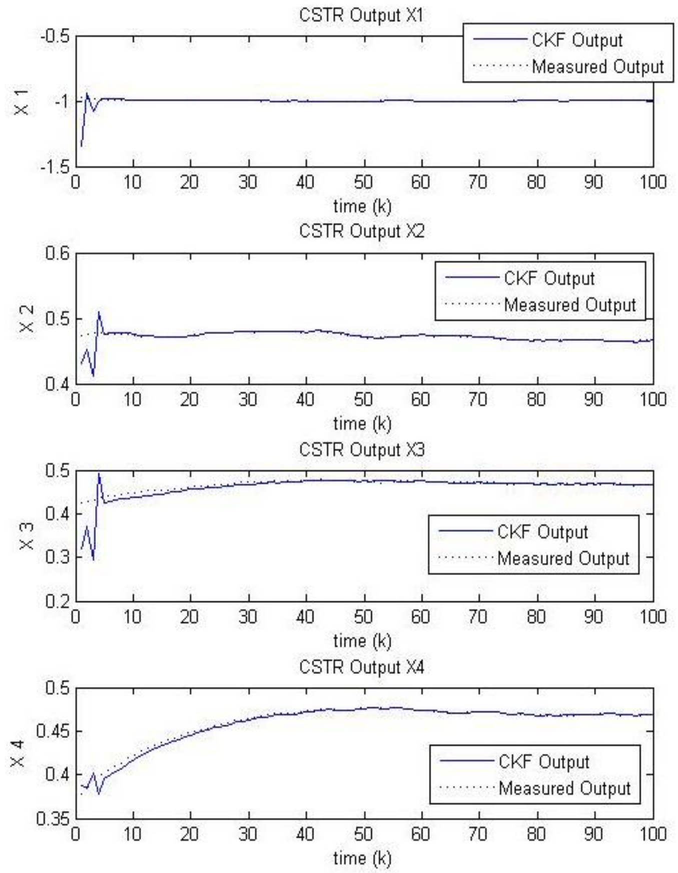

Figure 2: Estimated measurement with no fault (Scenario 1)

The measured filtered output is compared with expected fault trajectory to obtain the necessary error which can determined the similarity between the two comparisons. Figure 4 shows a set of errors detected for scenario 5, there are 8 expected trajectory generated to be compared with the measured signals. Each trajectory is associated with specific type of faults. Error 5 is related to combinations of fault actuator one and sensor one. The error that is very close to zero indicates the type of faults currently the systems is facing.

Table 2 shows the RMSE for the compared scenario and trajectories. Each expected fault trajectory or fault hypothesis is compared with the fault scenario. Comparisons between the two signals produce an error that is used to determine the fault. The lowest RMSE value in the scenario indicates the fault that the system is currently exposed. Large RMSE shows dissimilarity between the faults occurred in the measured data and expected fault trajectories. Thus, the fault can be deduced.

V. CONCLUSION

Identifying sensors and actuators fault can help user to focus on the source of the fault at the same time reduce the time and cost to mitigate the problem. In this paper, we proposed FDD sensor and actuator fault in nonlinear system.

The advantage of CKF is exploited in the proposed scheme and able to compensate any nonlinearities, measurement noise, and to differentiate sensor and actuator fault. Also, the CKF has been embedded in a multiple model scheme, resulting the multiple expected fault trajectory.

The number of models is based on the number of possible fault logic, in this case, the actual single sensor fault, single actuator fault or combination of faults can accurately be determined. Deducing the type of fault occur falls under classifying problem, there are many way of classification and can be the extension of this research. The CSTR simulation shows the effectiveness of the fault isolation using the proposed methods.

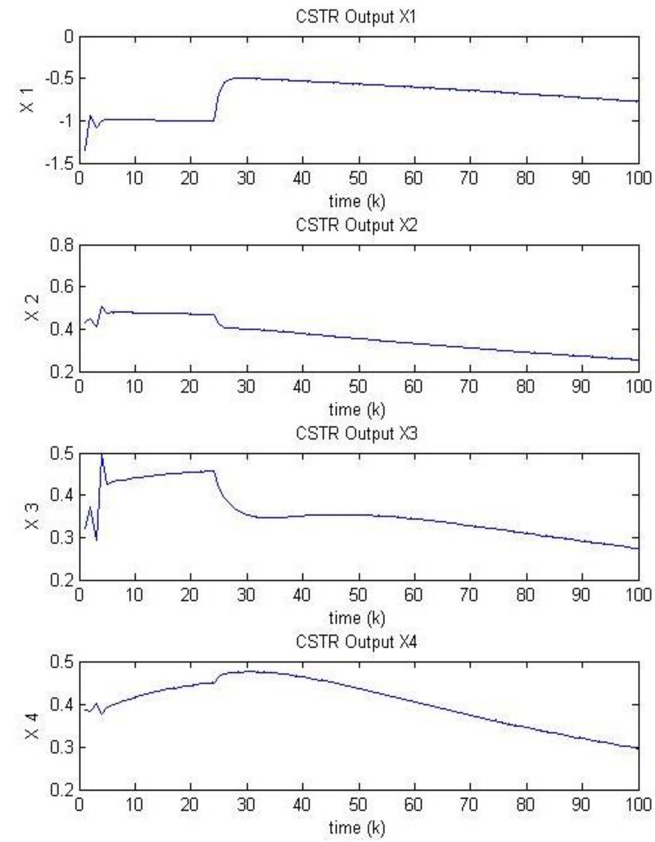

Figure 3: Estimated measurement with fault at time, k=25, (Scenario 5)

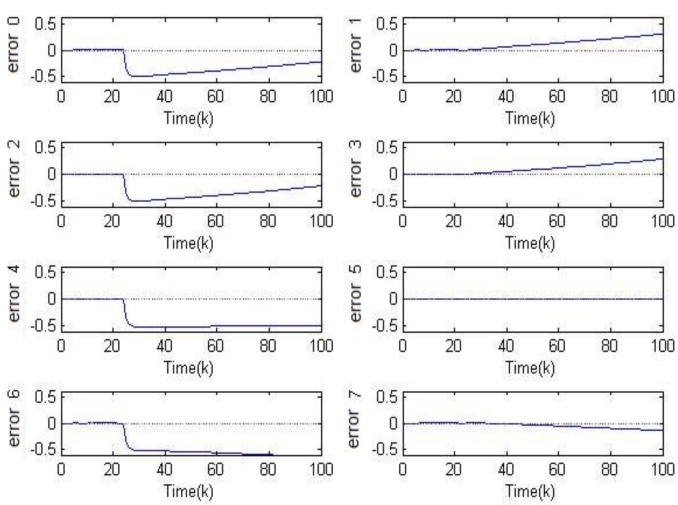

Figure 4: Generated possible expected trajectory error from Scenario 5 ($e_{5,i}$, $i = 0 \ldots 7$). Note $e_{5,5}$ have signal very close to zero to indicate similarity.

Table 2
RMSE Comparison Between Scenario and Expected Fault Trajectory

Fault Hypothesis	0	1	2	3	4	5	6	7
Scenario 1	0.140	40.970	18.771	44.438	15.786	33.659	27.980	37.858
Scenario 2	40.986	0.114	43.115	14.463	50.606	14.076	58.050	24.615
Scenario 3	19.288	43.249	1.050	38.448	12.273	30.100	15.299	24.779
Scenario 4	44.715	14.858	38.521	0.949	49.115	11.431	53.082	15.246
Scenario 5	16.016	51.319	12.282	49.578	0.583	39.522	12.335	36.989
Scenario 6	33.205	14.352	29.478	11.581	38.436	0.569	44.514	11.595
Scenario 7	28.633	58.571	16.207	53.665	13.363	45.531	1.079	38.743
Scenario 8	38.097	25.367	24.743	16.206	36.264	12.396	37.866	1.137

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