Modelling of Shre Drag Tilt Velocimeter (DTV) with Curvilinear, Gompertz and Artificial Neural Network Method

I.A.M Muharram[1], Z.H Ismail[2]
[1]*Department of Control and Mechatronics Engineering, Faculty of Electrical Engineering, Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia.*
[2]*Centre of Artificial Intelligence and Robotic, Universiti Teknologi Malaysia, Jalan Sultan Yahya Petra, Kuala Lumpur, Malaysia.*
ibnuakil.jkr@1govuc.gov.my

***Abstract*—Different method of modelling presented in this paper on Shre Drag Tilt Velocimeter non-linear data. The idea of different non-linear modelling method is to know which makes more possible to describe more accurate on interacting effects between velocities and tilt angle when compared among modellers. The models, which were used are static analytic approximation model, curvilinear bivariate regression model, Gompertz the classical growth model and Artificial Neural Network (ANN) model. Accuracy of the models was determined by mean square error (MSE), mean absolute deviation (MAD), bias and R Square. The datasets gathered from an experiment of Shre DTV at flume were divided into training data and testing data for the purpose of developing and validating all type of models. The difference between the model and the observed value become the forecasting error measurements. For the training data, the lowest MSE, RMSE and better R Square were noted for the Gompertz model. But ANN generalized better on testing data by obtaining lowest MSE, RMSE and higher R Square among others. ANN generalization result is 88.60%, Gompertz is 54.89%, curvilinear is 69.28% and static analytic is -1.29%. Lower bias was also for the neural network test data. As demonstrated by the bias values, only curvilinear model presenting overestimation model while other models produce little or no overestimation of the observed tilt response. Interpretations of the parameters estimation on Gompertz model have been attempted previously. However, focusing on the ability of Shre DTV to predict responses may be more practical than the relevance of parameter estimates.**

***Index Terms*—Drag Tilt; Modelling; Velocimeter.**

I. Introduction

The development of DTV has brought in the simplest technology and hydrological concepts into measuring the velocity of water at the river and streams. This innovative approach in developing an electronic flow meter is based on the drag force that a body will experience when immersed in a fluid stream and has proven excellent in measuring flow in the ocean for aquatic study [1]. This Shre DTV is meant to be deployed in the river as it is required for measuring water velocity in field for the mini hydro project. It will serve the purpose of judging the river conditions to locate river flows which will suit the turbine placement in order to assure a highly profit and low risk project development as well as boosting the investor's confidence [2].

Modelling is about abstracting a model that is valid, credible, feasible and useful and by this we mean a model that is fit-for-purpose. Modelling could be also a miniature representation of something like a pattern, example for imitation or emulation [3]. Given an ample data, it would be quite possible to build a simulation model that took account of all these details. Lack of collected data could make such a model infeasible [4]. Modelling can provide information that would give a greater level of confidence in making decision on describing on how devices or object of interest behave [5, 6]. There are several ways in which devices can be described such as using words, sketches or drawings, physical model, computer programs or mathematical formulas.

Another alternative to regression analysis for instrument modelling is by Neural Network. There is no published research has been conducted to model a DTV using neural network method. Ross Marchant et al. [1] compared the modelling of the data set of their Marotte DTV with analytic approximation and Gompertz growth model. They found that Gompertz produced models that adequately predicted the tilt. However, the neural network model was found to be superior in term of accuracy and precision [7, 8]. A comparison was made between the modelling by the analytic approximation, curvilinear, gompertz and neural network modelling in this study.

II. Literature review

Identification of real and conceptual worlds is depicted in Figure 1 below. In the real world, we observe various natural phenomena; whereas the conceptual world is where we try to understand what is going on therein. The conceptual world can be viewed as having three stages: observation, modelling and prediction [3]. In the observational part of the scientific method, we measure what is happening in the real world and collect empirical evidence. Observations may be direct, as when we use our senses; indirect, in which case some measurements indicate that an event has taken place through some other reading. The modelling part is concerned with analysing the above observations for one of (at least) three reasons [3]. The objective is to develop models that describe the observed behaviour or result; models that explain why that behaviour or result occurred as it did; and models that allow us to predict future behaviour or result that is yet to be seen or measured

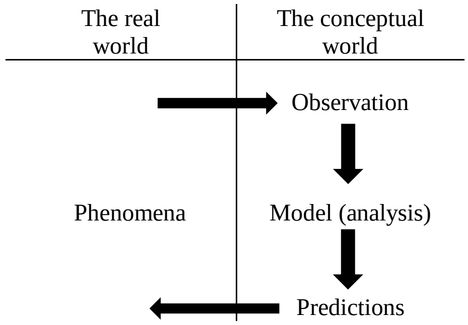


Figure 1: An elementary depiction of the scientific method of real and conceptual world [3]

Figure 2 exemplifies results from the Shre DTV model: tilt angle for velocity run. This example illustrates the very essence of conceptual modelling using the static analytic approximation model [9]; abstracting a model using hydrodynamic concept into the real system. In this case, the real system did not completely exist, but its simplifications helped the designer build the Shre DTV device instrument [9-11]. The static analytic model assumes that as velocity increases, the tube tilting achieves its saturated level at different stages depending on the k-value characteristic. Because of its constraints when looking into the dynamic behaviour of water, the conceptual model using static analytic approximation model might be described as a 'far abstraction'.

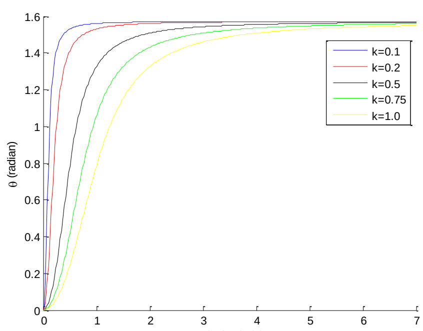


Figure 2: Tilt Angle for Velocity Run of Shre DTV

Since the aim of modelling is to identify the best possible model with the knowledge, time and resource available, the best conceptual model will be discovered when analysis has been done. Designing, building and fully testing all possible simulation models for a system and picking the best is the next further steps to be conducted. Method of doing comparative analysis among models always discovers the right and appropriate technique to be implemented on that system.

A. Type of Model

It is important to emphasize that a model is not the real world but merely a human construct to help us better understand on the real world systems that can come in many shapes, sizes, and styles. In general all models have an information input, an information processor, and an output of expected results [4].

B. Analytic Approximation Model

Analytic models are mathematical models that have a closed-form solution, which can be expressed as a mathematical analytic function. Some researchers argue that analytic models are more aesthetically pleasing since an inspection of the mathematical function can give information about the system's behaviour without the need for graphing or generating a table of values [12]. Analytical solutions to equations describing more complex systems can often become complicated. One disadvantage of analytical solutions is that they are mathematically challenging to obtain [13].

Analytic approximation model that was obtain by B.Kjerve [9] for drag-tilt behavior is:

$$v = k\left(\sqrt{\tan\theta}\right) \tag{1}$$

where:

k is sensitivity factor, (unitless)

θ is the tilt angle, (radian)

while:

$$k = \sqrt{\frac{2\, m_{sub}\, g}{\rho\, C_D\, A}} \tag{2}$$

Figure 2 shows how the drag-tilt behavior changes through velocity and its sensitivity over k-parameter values.

C. Curvilinear Model

Nonlinear data has many meanings, only some of which are (directly) about curves. Curvilinear model constructed to yield a smooth curve. For a graph shape that is depicted at Figure 2, it is curves. Curvilinear model constructed to yield a smooth curve [14]. For a graph shape that is depicted at Figure 2, it is shown that the curve for predicting *y* from *x* is a negatively accelerated curve. A curve of decreasing returns; where the positive slope decreases as *x* increases. A polynomial regression may do a better job to obtain less error than does the linear model when dealing with Shre DTV nonlinear data. The polynomials basic function for cubic type was selected since the slope for predicting *y* from *x* changes direction twice as per below:

$$y\,(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \tag{3}$$

D. Gompertz Model

Gompertz model or Gompertz curve, named after Mr Benjamin Gompertz, is an S-type, symmetrical and sigmoid function which was introduced in 1825 [15]. Its auxiliary names were labelled by former researches from their respective fields. This model is mathematically flexible. The Gompertz curve has its own characteristic which can be observed clearly by looking at the inflection point, the rate of change, and the saturation level. These characteristics are very powerful in inferring data or system, and are also useful as predicting tools when fitting Gompertz growth curves.

The mathematical representation of Gompertz process y (x), is given by:

$$y\ (x) = \mathrm{a}\ exp^{-b\ exp^{-c\,(x)}} \qquad (4)$$

where a,b,c are unknown positive-valued parameters. Any application that using Gompertz model, agree that point of inflection is the point where the rate of growth changes from increasing to decreasing [16-18].

E. ANN Model

An ANN is a massive parallel-distributed processor that has a natural propensity for storing the experimental knowledge and making it available for future use [19] . It resembles the human brain in speed and efficiency. The quest to understand processes and to solve the associated problems has led to the development of the ANN technique. Neural networks essentially involve a nonlinear modelling approach that provides a fairly accurate universal approximation for any function. Its power comes from the parallel processing of information data [20]. No prior assumption of the model form is required in the modelling process. Instead, the network model is largely determined by the data characteristic [21].

The Feed Forward Multi-Layer Perceptron (MLP) Neural Networks consists of at least three layers: input, output and hidden [19] . The number of input and output neurons in the input and output layers is determined by the conditions of the evaluated problem. Often, the number of hidden neurons and hidden layers is selected by a tedious trial and error method [22]. Figure 3 shows the structured model of a neuron. The neuron inputs are transferred to a collector by multiplying them with weights on the synaptic bonds. The outputs are calculated by passing the sum from the activation function of the neuron.

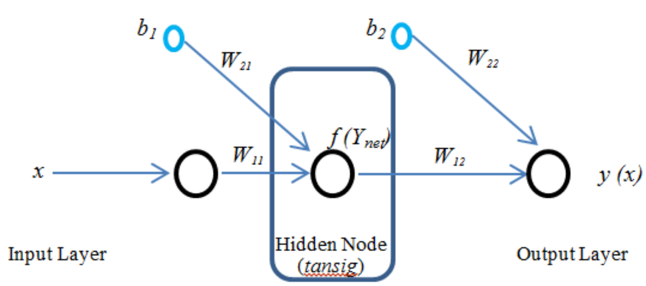


Figure 3: Neural Network Structured Model

$$y(x) = W_{12}\, f(x\, W_{11} + b_1\, W_{21}) + \ b_2 W_{22} \qquad (5)$$

$$f(x\, W_{11} + b_1\, W_{21}) = f\ (Y_{net}) \qquad (6)$$

y(x) is the result of the neural network, $f\ (Y_{net})$ is the activation function with (Y_{net}) as the summation function. *x* is the input variables, W_{ii} is the weight coefficient of each input neuron and b_i is the bias.

F. Measurement of fitness

Quantitative measurement of the fit of the predictive models which were commonly used to evaluate models was made using error measurement indices [23-24]. The accuracy of models was made using:

i) Mean Absolute Deviation (MAD), computed as;

$$\mathrm{MAD} = \frac{\sum_{t=1}^{n} |y_t - \widehat{y_t}|}{n} \qquad (7)$$

where y_t equals the observed value at time t, $\widehat{y_t}$ equals the estimated value, and n equals the number of observations;

ii) Mean Square Error (MSE), computed as;

$$\mathrm{MSE} = \frac{\sum_{t=1}^{n} |y_t - \widehat{y_t}|^2}{n} \qquad (8)$$

where y_t equals the observed value at time t, $\widehat{y_t}$ equals the estimated value, and n equals the number of observations; and

iii) Bias, computed as;

$$\mathrm{BIAS} = \frac{\sum_{t=1}^{n} y_t - \widehat{y_t}}{n} \qquad (9)$$

where y_t equals the observed value at time t, $\widehat{y_t}$ equals the estimated value, and n equals the number of observations.

III. DATASET FRAMEWORK

The Shre DTV calibrated at National Hydraulic Research Institute Malaysia (NAHRIM), Seri Kembangan, Selangor. There have 60 long flume which can operate reliably at lower speed over a velocity range of 0-0.7ms^{-1}. The flume section is 60 meters long, 2 meters deep and 1.2 meters wide. It has a combination of concrete walls and glass side wall. Figure 4 below shows the plan view diagram of the experimental set up for this calibration process.

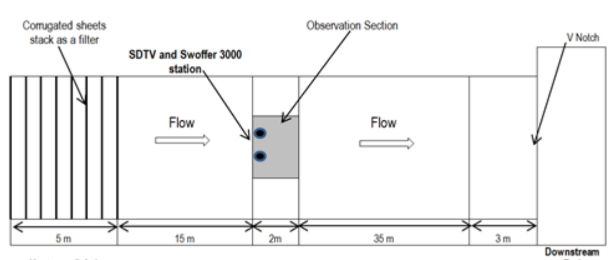


Figure 4: Plan view of the experimental set up

To perform the experiment, the flume was filled with water up to 1.2 meters deep. The flow was generated by circulating the flow from the holding pond at the downstream end of the flume back to the upstream end of the flume. To accomplish the process, 2 units of variable heavy duty electrical powered 4 hp pumps were installed to force the water up with the pump inlet connected to the downstream holding pond then the pump outlet was connected to an inlet of the upstream flume structure which goes through a series of filters. The entrance to the flume was designed to give disturbance free uniform velocities by series of filters so fairly uniform laminar flows go through the flume.

A. Experiment data

The input variable for the flow experiments was the velocity of the water, which was varied from 0 m/s to 0.7 m/s. The main output variable was the tilt angle of the tube, which was measured by the accelerometer in radian readings at the same time water speed was observed.

There are 36 data measurements set have been taken as per Table 1 below and used as the training data. While Table 2 consists of 28 data measurement utilize as testing data.

Table 1
Training Data

No.	Velocity (m/s)	Tilt (radian)	No.	Velocity (m/s)	Tilt (radian)
1	0	0	19	0.46	0.625829
2	0.11	0.040231	20	0.47	0.433345
3	0.16	0.060591	21	0.48	0.495160
4	0.20	0.238770	22	0.49	0.366550
5	0.23	0.154509	23	0.50	0.593315
6	0.24	0.179153	24	0.51	0.625829
7	0.26	0.157985	25	0.52	0.413294
8	0.28	0.337778	26	0.53	0.688031
9	0.31	0.147055	27	0.54	0.517003
10	0.35	0.391214	28	0.55	0.581708
11	0.36	0.366550	29	0.56	0.582107
12	0.38	0.495160	30	0.57	0.558471
13	0.39	0.625829	31	0.58	0.560022
14	0.40	0.527981	32	0.59	0.560802
15	0.41	0.502350	33	0.61	0.609169
16	0.42	0.430020	34	0.62	0.603616
17	0.44	0.350036	35	0.63	0.657536
18	0.45	0.430855	36	0.64	0.668106

Table 2
Test Data

No.	Velocity (m/s)	Tilt (radian)	No.	Velocity (m/s)	Tilt (radian)
1	0.16	0.029889	15	0.11	0.003322
2	0.11	0.053102	16	0.11	0.013287
3	0.11	0.013287	17	0.16	0.029889
4	0.11	0.003322	18	0.11	0.003322
5	0.11	0.003322	19	0.11	0.029889
6	0.11	0.053102	20	0.16	0.029889
7	0.11	0.029889	21	0.35	0.560802
8	0.2	0.161363	22	0.39	0.625829
9	0.28	0.366550	23	0.2	0.305887
10	0.16	0.082860	24	0.41	0.625829
11	0.16	0.082860	25	0.41	0.625829
12	0.11	0.053102	26	0.23	0.249061
13	0.24	0.196942	27	0.2	0.249061
14	0.11	0.003322	28	0.28	0.430019

IV. RESULTS AND INTERPRETATION

The following Analytic, Curvilinear, Gompertz and ANN equation were identified as fit for the data:

(i) Static analytic

$$y\,(x) = \mathrm{atan}\,\left({}^{x}/_{0.694}\right)^{2}$$

(ii) Curvilinear

$$y(x) = 0.021792 + 0.34766x + 3.5009x^2 + 3.9033x^3$$

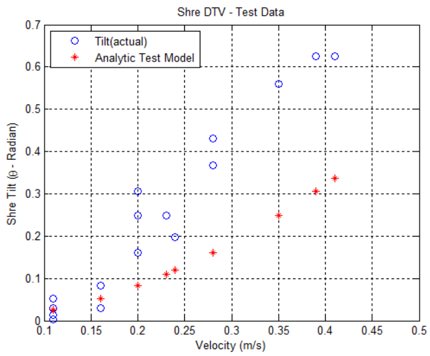
(iii) Gompertz

$$y\,(x) = 0.6699\;exp^{-5.9447\;exp^{-6.6511(x)}}$$

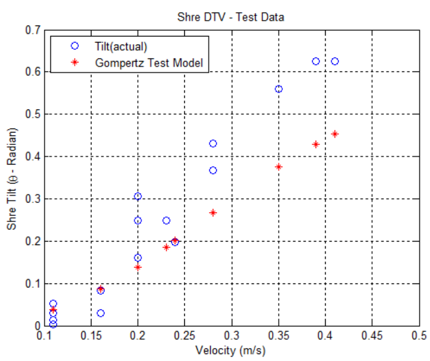
(iv) ANN

$$y(x) = W_{12}\;tansig(x\,W_{11} + b_1\,W_{21}) + \;b_2W_{22}$$

Figures 5(a) and 5(b) and Table 3 show the resulting test for each modeling method. Forecasting error measurements based on the difference of model and the observed values are shown. ANN model consistently produced the lowest MSE while Gompertz model have lowest MAD. Only Curvilinear model shows overestimation while the other three models does not show any overestimation of the tilt response as the bias value shows relatively low. Table 4 lists the statistic for all models resulting from the training and testing data.



(a) Analytical Model



(b) Gompertz Model

Figure 5: Visual Depiction of Test Data Model by all Modeling Technique

Table 3
Details of Modelling Error Evaluations on Test Data

Model	MSE	MAD	BIAS
Static Analytic Approximation	0.0214	0.0974	0.0829
Curvilinear	0.0242	0.1259	-0.1228
Gompertz	0.0083	0.0650	0.0348
Artificial Neural Network	0.0017	0.0754	0.0181

Table 4
Details of Modelling Result on Training and Test Data

	Test Data			Training Data			
Model	MSE	RMSE	R-Sq %	MSE	RMSE	R-Sq %	Hidden Neurons
Static Analytic Approximation	0.0063	0.079	84.79	0.0214	0.146	-1.294	-
Curvilinear	1.1201	1.058	98.90	0.0242	0.156	69.28	-
Gompertz	0.0003	0.017	98.93	0.0083	0.091	54.89	-
Artificial Neural Network	0.0068	0.083	81.59	0.0017	0.041	88.60	1

V. CONCLUSION

The entire models have made their own curve fitting as well as the function approximation. Neural networks are quite often used to approximate complex mathematical functions and their performance found better than other modelling technique particularly for this Shre DTV nonlinear data. The structures of ANN, which consist of hidden layer extract salient features in the input data which have predictive power with respect to the output. The advantage of neural network is that there is no requirement of preselecting a model to fit the data while the disadvantage is that they take a 'black box' approach, which no clear picture of how the internal networks working. Further research on how the ANN neurons arranged by their connections between one unit and another will be focus in order to optimize the modelling technique using neural network. The practicality of using ANN method to predict response of Shre DTV are right in place for having the right model towards accurate measurement of the instruments.

REFERENCES

[1] R. Marchant, S. Member, T. Stevens, S. Choukroun, G. Coombes, M. Santarossa, J. Whinney, and P. Ridd, "A Buoyant Tethered Sphere for Marine Current Estimation," *IEEE J. Ocean. Eng.*, vol. 39, no. 1, pp. 2–9, 2014.

[2] JKR Malaysia and UNIMAS, "Energy Mapping Study at Schools along Sungai Balleh, Kapit, Sarawak.," 2010.

[3] P. E. Wellstead, *Introduction to Physical System Modelling*, 1979th ed. Kent, Great Britain: Control Systems Principles, (www.control-systems-principles.co.uk), 1979.

[4] M. M. Rosli, E. Tempero, and A. Luxton-reilly, "Can We Trust Our Results ? A Mapping Study on Data Quality," in *20th Asia-Pacific Software Engineering Conference*, 2013, pp. 116–23.

[5] Y. I. Shtern, Y. S. Kozhevnikov, V. M. Rykov, and R. E. Mironov, "Mathematical Models and Hardware & Software for Highly Accurate Electronic Temperature Meters," *Russ. Microelectron.*, vol. Volume 42, no. 7, pp. 384–388, 2013.

[6] H. Cheng, Y. Zhao, Z. Qian, and B. Li, "Research on Performance Analyzing and Modeling of Quartzose Flexible Accelerometer," in *The Ninth International Conference on Electronic Measurement & Instruments (ICEMI 2009)*, 2009, pp. 412–415.

[7] S. L. B. W.B. Roush, W.A. Dozier III, "Comparison of gompertz and neural network models of broiler chickens," *Poult. Sci.*, vol. 85, pp. 794–797, 2006.

[8] M. Xiao, P. Huang, P. Lin, and S. Yan, "Broiler Growth Performance Analysis :from Correlation Analysis , Multiple Linear Regression , to Neural Network," no. 2007, pp. 25–28, 2010.

[9] B. Kjerfve, "Calibration of estuarine current crosses," *Estuar. Coast. Shelf Sci.*, vol. 15, no. 5, pp. 553–559, 1982.

[10] Z. H. Ismail and I. A. M. Muharram, "Design Consideration of Drag Tilt Velocimeter for River Flow Studies," in *The 3rd International Conference on Water Resources ICWR-2015*, 2015, no. 2015, pp. 1–10.

[11] Z. H. Ismail and I. A. M. Muharram, "Calibration of a Drag Tilt Current Velocimeter for an Open Channel Application," *J. Teknol. UTM*, vol. 78, no. 6–13, pp. 89–94, 2016.

[12] M. Azarkhail, "Model for Treatment of Incomplete Records in Preventive Maintenance," in *IEEE Conference Publication- Reliability and Maintainability Symposium (RAMS), 2013 Proceedings*, 2013, pp. 1–7.

[13] B. J. Baliga, "Analytical Modeling of IGBTs : Challenges and Solutions," *IEEE Trans. Electron Devices*, vol. 60, no. 2, pp. 535–543, 2013.

[14] R. G. L. Randall E. Schumacker, *A Beginner's Guide to Structural Equation Modeling*, 2nd Editio. Lawrence Erlbaum Associates, Inc, 2005.

[15] C. P. Winsor, "The Gompertz Curve as a Growth Curve.," *Proc Natl Acad Sci U S A*, vol. 18, no. 1, pp. 1–8, 1932.

[16] M. Pitchaimani and T. Eakin, "Unique estimation of Gompertz

parameters with mortality deceleration rate," vol. 47, pp. 104–114, 2008.

[17] C. F. Ã. Lo, "Stochastic Gompertz Model of Tumour Cell Growth," *J. Theor. Biol.*, vol. 248, pp. 317–321, 2007.

[18] X. Li and E. Wang, "Prediction of Electric Vehicle Ownership Based on Gompertz Model *," in *Proceeding of the IEEE International Conference on Information and Automation*, 2014, no. July, pp. 87–91.

[19] S. Haykin, *Neural Networks: A Comprehensive Foundation*. MacMillan Publishing Company, 1994.

[20] Q. Dai, S. Xu, and X. Li, "Parallel Process Neural Networks and its Application in the Predication of Sunspot Number Series," in *2009 Fifth International Conference on Natural Computation*, 2009, pp. 237–241.

[21] L. Yu, S. Wang, and K. K. Lai, "An Integrated Data Preparation Scheme for Neural Network Data Analysis," *IEEE Trans. Knowl. Data Eng.*, vol. 18, no. 2, pp. 1–14, 2006.

[22] N. Clarence and W. Tan, *An Artificial Neural Networks Primer with Financial Applications Examples in Financial Distress Predictions and Foreign Exchange Hybrid Trading System ' by*. School of Information Technology, Bond University, Gold Coast, QLD 4229, Australia, 1999.

[23] Oberstone, *Management Science – Concepts, Insights, and Applications*. West Publication Co. New York, 1990.

[24] J. R. Taylor, *An Introduction to Error Analysis-The Study of Uncertainties in Physical Measurement.* University Science Books, 1982.