

# THE CONJUGACY CLASSES OF SOME FINITE METABELIAN GROUPS AND THEIR RELATED GRAPHS

Nor Haniza Sarmin<sup>a\*</sup>, Ain Asyikin Ibrahim<sup>a</sup>, Alia Husna Mohd Noor<sup>a</sup>, Sanaa Mohamed Saleh Omer<sup>b</sup>

<sup>a</sup>Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia

<sup>b</sup>Department of Mathematics, Faculty of Science, University of Benghazi, Benghazi, Libya

## Article history

Received

28 February 2016

Received in revised form

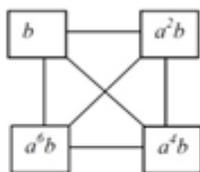
16 September 2016

Accepted

10 December 2016

\*Corresponding author

nhs@utm.my



$K_4$

## Abstract

In this paper, the conjugacy classes of three metabelian groups, namely the Quasi-dihedral group, Dihedral group and Quaternion group of order 16 are computed. The obtained results are then applied to graph theory, more precisely to conjugate graph and conjugacy class graph. Some graph properties such as chromatic number, clique number, dominating number and independent number are found.

**Keywords:** Metabelian group, conjugacy class, conjugacy class graph, conjugate graph, properties of graph

## Abstrak

Dalam kertas kerja ini, kelas konjugat bagi tiga kumpulan metabelan, iaitu Quasi-dihedral, kumpulan Dihedral dan kumpulan berperingkat 16 dikira. Keputusan yang diperolehi ini kemudiannya diaplikasikan ke dalam graf teori, lebih tepat kepada graf konjugat dan graf kelas konjugat. Beberapa sifat graf seperti nombor kromatik, nombor kelompok, nombor dominasi dan nombor bebas diperolehi.

**Kata kunci:** Kumpulan metabelan, kelas konjugat, graf kelas konjugat, graf konjugat, sifat graf

© 2017 Penerbit UTM Press. All rights reserved

## 1.0 INTRODUCTION

A metabelian group is a group  $G$  that possesses at least a normal subgroup  $N$  such that  $N$  and  $G/N$  are both abelian. A group  $G$  is also called metabelian if its commutator is abelian [1]. Every abelian group is metabelian. However, not every metabelian group is abelian. In 2012, Rahman *et al.* are studied on metabelian groups of order at most 24. From this paper, they have found 59 metabelian groups of order less than 24. The results showed that 34 groups of them are abelian while the rest were considered

as non-abelian groups [2].

The conjugate graph is a graph whose vertices are non-central elements of  $G$ , in which two vertices are adjacent if they are conjugate. The conjugacy class graph is a graph whose vertices are non-central conjugacy classes of a group  $G$  in which two vertices are connected if their cardinalities are not coprime.

Study on graphs related to groups has recently become very popular to many authors. In this paper, the number of conjugacy classes of the groups mentioned earlier is computed and the results are then applied to some related graphs.

This paper is divided into three sections. The first section is the introduction and layout of the problem, while the second section provides some basic definitions on group theory and graph theory, also some earlier and recent publications that are related to conjugacy classes, conjugate graph and conjugacy class graph. In the third section, we introduce our main results, which include the conjugacy classes of three metabelian groups of order 16, their conjugate graph and conjugacy class graph.

## 2.0 METHODOLOGY

This section provides some background related to group theory and graph theory. Some basic concepts of group theory that are needed in this paper are stated starting with the definition of a conjugacy class. The conjugacy class is an equivalence relation, in which the group is partitioned into disjoint sets.

The following definitions and propositions are used to compute the conjugacy classes of three metabelian groups of order 16, namely the Quasi-dihedral group, Dihedral group and Quaternion group of order 16.

### Definition 2.1. [3]

Let  $G$  be a finite group and  $x, y \in G$ . The elements  $x, y$  are said to be conjugate if there exists  $g \in G$  such that  $y = gxg^{-1}$ . The set of all conjugates of  $x$  is called the conjugacy classes of  $x$ .

### Definition 2.2 [3]

Let  $G$  be a finite group. The conjugacy class of the element  $a$  in  $G$  is given as:

$$cl(a) = \{g \in G : \text{there exist } x \in G \text{ with } g = xax^{-1}\}.$$

In this paper,  $K(G)$  denotes the number of conjugacy classes of  $G$ . All elements belong to the same conjugacy class have the same order. Thus, the identity element is always in its own class, that is  $cl(e) = \{e\}$ .

**Proposition 2.1.** [3] Suppose that  $G$  is a finite group where  $a$  and  $b$  are the elements of  $G$ . The elements  $a$  and  $b$  are called conjugate if they belong to one conjugacy class, that is  $cl(a)$  and  $cl(b)$  are equal.

Some basic concepts of graph theory that are needed in this paper are given next.

### Definition 2.3. [4]

A graph  $\Gamma$  is a mathematical structure consisting of two sets namely vertices and edges denoted by  $V(\Gamma)$  and  $E(\Gamma)$ , respectively.

### Definition 2.4. [4]

A subgraph of a graph  $\Gamma$  is a graph whose vertices and edges are subset of the vertices and edges of  $\Gamma$ . Hence we denote  $\Gamma_{sub}$  a subgraph of  $\Gamma$ .

### Definition 2.5. [5]

A complete graph is a graph where each ordered pair of distinct vertices are adjacent denoted by  $K_n$ .

### Definition 2.6. [6]

A non-empty set  $S$  of  $V(\Gamma)$  is called an independent set of  $\Gamma$  if there is no adjacent between two elements of  $S$  in  $\Gamma$ . This can be obtained by  $K(G) - |Z(G)|$ , where  $Z(G)$  is the center of  $G$ .

### Definition 2.7. [6]

The independent number is the number of vertices in maximum independent set and it is denoted by  $\alpha(\Gamma)$ .

### Definition 2.8. [4]

The minimum number  $c$  for which  $\Gamma$  is  $c$ -vertex colorable is known as the chromatic number and it is denoted by  $\chi(\Gamma)$ .

### Definition 2.9. [4]

Clique is a complete subgraph in  $\Gamma$ . The size of the largest clique in  $\Gamma$  is called the clique number and is denoted by  $\omega(\Gamma)$ .

### Definition 2.10. [6]

The dominating set  $X \subseteq V(\Gamma)$  is a set where for each  $v$  outside  $X$ ,  $\exists x \in X$  such that  $v$  is adjacent to  $x$ . This can be obtained by  $K(G) - |Z(G)|$ , where  $Z(G)$  is the center of  $G$ . Thus, minimum size of  $X$  is called the dominating number and it is denoted by  $\gamma(\Gamma)$ .

The following are the definitions of conjugate graph and conjugacy class graph.

### Definition 2.11. [6]

Suppose  $G$  is a finite group with  $Z(G)$  as the center of  $G$ . The vertices of a conjugate graph of  $G$ , denoted by  $\Gamma_G^c$ , are the non-central elements of  $G$ , that is  $|V(\Gamma_G^c)| = |G| - |Z(G)|$  in which two vertices are adjacent if they are conjugate.

### Definition 2.12. [7]

Suppose  $G$  is a finite group with  $Z(G)$  as the center of  $G$ . The vertices of the conjugacy class graph of  $G$ , denoted by  $\Gamma_G^{cl}$ , are non-central conjugacy classes of  $G$  for which  $|V(\Gamma_G^{cl})| = K(G) - |Z(G)|$ , where  $K(G)$  is the number of conjugacy classes in  $G$ . Two vertices are adjacent if their cardinalities are not coprime, that is, the greatest common divisor of the number of vertices is not equal to one.

Next, some earlier studies on conjugacy classes, conjugate graph and conjugacy class graph are given.

The concept of conjugacy classes is widely used by many researches. The conjugacy classes have been used in several concepts including the commutativity degree [8], the probability that a

group element fixes a set [9] and many others. The concept of conjugacy classes is also associated to graph theory where the sizes of conjugacy classes are used.

In 1990, Bertram *et al.* [7] introduced a graph called the conjugacy class graph. The vertices of this graph are non-central conjugacy classes, where two vertices are adjacent if their cardinalities are not coprime. As a consequence, numerous works have been done on this graph and many results have been achieved.

In 2005, Moreto *et al.* [10] studied the diameter of the conjugacy class graph in which a group satisfies the property  $P_n$  that is for all prime integer  $p$ ,  $G$  has at most  $n - 1$  conjugacy class whose size is multiple of  $p$ . Recently, Bianchi *et al.* [11] studied the regularity of the graph related to conjugacy classes and provided some results. In 2013, Ilangovan and Sarmin [12] found some graph properties of graph related to conjugacy classes of two-generator two-groups of class two. In the same year, Moradipour *et al.* [13] used the graph related to conjugacy classes to find some graph properties of some finite metacyclic 2-groups.

Much later, Erfanian and Tolue [6] introduced a new graph which is called a conjugate graph. This conjugate graph is associated to a non-abelian group  $G$  with vertex set  $G/Z(G)$  such that two distinct vertices join by an edge if they are conjugate.

### 3.0 RESULTS AND DISCUSSION

This section consists of two parts. The first part focuses on finding the conjugacy classes of some metabelian groups of order 16, while the second part applies the obtained results to the graph theory, specifically to conjugate graph and conjugacy class graph

#### 3.1 The Conjugacy Classes of Some Metabelian Groups of Order 16

In this section, the conjugacy classes of some metabelian groups of order 16 are computed, starting with the dihedral group of order 16, denoted as  $D_8$ .

**Theorem 3.1.** Let  $G$  be a dihedral group of order 16,  $G \cong D_8 \cong \langle a, b \mid a^8 = b^2 = e, bab^{-1} = a^{-1} \rangle$ . Then, the number of conjugacy classes of  $D_8$  is  $K(D_8) = 7$ .

*Proof.* The elements of  $D_8$  are listed as follows:  $D_8 = \{e, a, a^2, a^3, a^4, a^5, a^6, a^7, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b\}$ . By using Definition 2.1 and Definition 2.2 where  $cl(x) = \{gxg^{-1} : g \in G\}$ , the conjugacy classes of  $G$  are determined as follows: Let  $x = e$ , then  $cl(e) = \{e\}$ . Next, let  $x = a$ , then  $cl(a) = \{gag^{-1}, \text{ for all } g \in D_8\}$ . Thus  $cl(a) = \{a, a^7\}$  using Proposition 2.1, then  $cl(a) = cl(a^7)$ . Using the same

procedure, the conjugacy classes of  $G$  are found and listed as follows:  $cl(e) = \{e\}$ ,  $cl(a) = \{a, a^7\}$ ,  $cl(a^2) = \{a^2, a^6\}$ ,  $cl(a^3) = \{a^3, a^5\}$ ,  $cl(a^4) = \{a^4\}$ ,  $cl(b) = cl(a^6b) = \{b, a^2b, a^4b, a^6b\}$ ,  $cl(ab) = \{ab, a^3b, a^5b, a^7b\}$ . Therefore,  $K(D_8) = 7$ , as claimed. ■

Next, the conjugacy classes of quasi-dihedral group of order 16 are found.

**Theorem 3.2.** Let  $G$  be a quasi-dihedral group of order 16,  $G \cong Qh_{16} \cong \langle a, b \mid a^8 = b^2 = 1, bab = a^3 \rangle$ . The number of conjugacy classes of  $Qh_{16}$  is  $K(Qh_{16}) = 7$ .

*Proof.* First the elements of  $G$  are given as:  $G = \{e, a, a^2, a^3, a^4, a^5, a^6, a^7, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b\}$ . By using Definition 2.2, the conjugacy classes of  $G$  are determined. First let  $x = e$ , thus,  $cl(e) = \{e\}$ . Next let  $x = a$ , then  $cl(a) = \{gag^{-1}, g \in G\}$ . Thus  $cl(a) = \{a, a^9\}$ . By using Proposition 2.1, then  $cl(a) = cl(a^3)$ . Using the same procedure, the conjugacy classes of  $G$  are found as follows:  $cl(e) = \{e\}$ ,  $cl(a) = \{a, a^3\}$ ,  $cl(a^2) = \{a^2, a^6\}$ ,  $cl(a^4) = \{a^4\}$ ,  $cl(a^5) = \{a^5, a^7\}$ ,  $cl(b) = \{b, a^2b, a^4b, a^6b\}$  and  $cl(ab) = \{ab, a^3b, a^5b, a^7b\}$ . Thus the number of conjugacy classes in  $G$  is equal to seven, i.e.  $K(G) = 7$ . ■

**Theorem 3.3.** Let  $G$  be a quaternion group of order 16,  $Q_8$ , where the representation is  $Q_8 = \langle a, b \mid a^8 = 1, a^4 = b^2, bab^{-1} = a^{-1} \rangle$ . The number of conjugacy classes of  $Q_8$  is  $K(Q_8) = 7$ .

*Proof.* First, the elements of  $G$  are given as:  $Q_8 = \{e, a, a^2, a^3, a^4, a^5, a^6, a^7, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b\}$ . By using Definition 2.2, the conjugacy classes of  $G$  are determined. First let  $x = e$ , thus,  $cl(e) = \{e\}$ . Next let  $x = a$ , then  $cl(a) = \{gag^{-1}, g \in G\}$ . Thus  $cl(a) = \{a, a^7\}$ . By Proposition 2.1,  $cl(a) = cl(a^7)$ . By using the same procedure, the conjugacy classes of  $G$  are computed as follows:  $cl(e) = \{e\}$ ,  $cl(a) = \{a, a^7\}$ ,  $cl(a^2) = \{a^2, a^6\}$ ,  $cl(a^3) = \{a^3, a^5\}$ ,  $cl(a^4) = \{a^4\}$ ,  $cl(b) = cl(a^6b) = \{b, a^2b, a^4b, a^6b\}$  and  $cl(ab) = \{ab, a^3b, a^5b, a^7b\}$ . Thus the number of conjugacy classes of  $G$  is 7. ■

#### 3.2 The Conjugate Graph and Conjugacy Class Graph of Some Metabelian Groups of Order 16.

This section focuses on applying the obtained results in Section 3.1 to give some properties of conjugate graph and conjugacy class graph of the groups  $D_8$ ,  $Q_8$  and  $Qh_{16}$ . We begin with the conjugate graph.

### 3.2.1 The Conjugate Graph of Some Nonabelian Metabelian Groups of Order 16

The conjugate graph of some nonabelian metabelian groups of order 16 is determined, where the conjugate elements of each groups are used to determine the conjugate graph.

**Theorem 3.4.** Let  $G$  be a dihedral group of order 16,  $G \cong D_8 = \langle a, b \mid a^8 = b^2 = 1, bab = a^{-1} \rangle$ . Then,  $\Gamma_{D_8}^C = \bigcup_{i=1}^3 K_2 \cup \bigcup_{i=1}^2 K_4$ .

*Proof.* Based on Theorem 3.1, there are fourteen non-central elements in  $G$ , thus  $|V(\Gamma_{D_8}^C)| = 16 - 2 = 14$ . By vertices adjacency of conjugate graph and Proposition 2.1, the elements are conjugate if they belong to one conjugacy class. Thus  $\Gamma_{D_8}^C$  consists of three complete components of  $K_2$  and two complete components of  $K_4$ . Hence,  $\Gamma_{D_8}^C = \bigcup_{i=1}^3 K_2 \cup \bigcup_{i=1}^2 K_4$ . The conjugate graph of  $D_8$ ,  $\Gamma_{D_8}^C$  is shown in Figure 1. ■

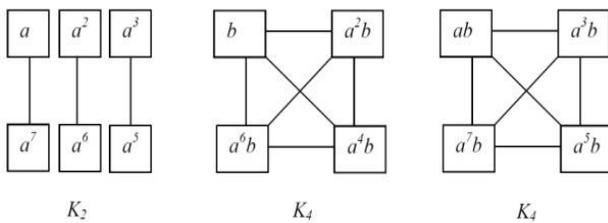


Figure 1 The conjugate graph of  $D_8$

According to Theorem 3.4, some graph properties can be found, given in the following.

**Corollary 3.1.** Let  $G$  be a dihedral group of order 16,  $G \cong D_8 = \langle a, b \mid a^8 = b^2 = 1, bab^{-1} = a^{-1} \rangle$ . Then  $\chi(\Gamma_{D_8}^C) = \omega(\Gamma_{D_8}^C) = 4$  and  $\alpha(\Gamma_{D_8}^C) = \gamma(\Gamma_{D_8}^C) = 5$ .

*Proof.* Based on Theorem 3.4 and Figure 1, there are three complete components of  $K_2$  and two complete components of  $K_4$ . Thus by Definition 2.8, the chromatic number is equal to four since the four vertices can be colored by four different colors. By Definition 2.9, the clique number,  $\omega(\Gamma_{D_8}^C) = 4$  since the largest subgraph in  $\Gamma_{D_8}^C$  is  $K_4$ . By Definition 2.7, the independent number of  $\Gamma_{D_8}^C$  is equal to 5 since the maximum independent set is  $\{a, a^2, a^3, b, ab\}$  means that there are five vertices that are not adjacent. By Definition 2.10, the  $\Gamma_{D_8}^C$  has the dominating number,  $\gamma(\Gamma_{D_8}^C) = 5$  since there are five vertex needed so that all the vertices in  $\Gamma_{D_8}^C$  will be connected.

**Theorem 3.5.** Let  $G$  be a quaternion group of order 16,  $G \cong Q_8 = \langle a, b \mid a^8 = 1, a^4 = b^2, bab^{-1} = a^{-1} \rangle$ . Then,  $\Gamma_{Q_8}^C = \bigcup_{i=1}^3 K_2 \cup \bigcup_{i=1}^2 K_4$ .

*Proof.* Based on Theorem 3.2,  $Z(Q_8) = \{1, a^4\}$ . Since the vertices of conjugate graph are non-central elements, thus  $|V(\Gamma_{Q_8}^C)| = 16 - 2 = 14$ . Since two vertices are adjacency if they are conjugate and using Proposition 2.1, thus  $\Gamma_{Q_8}^C$  consists of three complete components of  $K_2$  and two complete components of  $K_4$ . Hence,  $\Gamma_{Q_8}^C = \bigcup_{i=1}^3 K_2 \cup \bigcup_{i=1}^2 K_4$ . The conjugate graph of  $Q_8$  turns out to be the same as in Figure 1. ■

Based on Theorem 3.5, the following corollary is concluded.

**Corollary 3.2.** Let  $G$  be a quaternion group of order 16,  $G \cong Q_8 = \langle a, b \mid a^8 = 1, a^4 = b^2, aba = b \rangle$  and  $\Gamma_{Q_8}^C = \bigcup_{i=1}^3 K_2 \cup \bigcup_{i=1}^2 K_4$ . Then,  $\chi(\Gamma_{Q_8}^C) = \omega(\Gamma_{Q_8}^C) = 4$  and  $\alpha(\Gamma_{Q_8}^C) = \gamma(\Gamma_{Q_8}^C) = 5$ .

**Theorem 3.6.** Let  $G$  be a Quasi-dihedral group of order 16,  $G \cong Qh_{16} = \langle a, b \mid a^8 = b^2 = 1, bab = a^3 \rangle$ . Then,  $\Gamma_{Qh_{16}}^C = \bigcup_{i=1}^3 K_2 \cup \bigcup_{i=1}^2 K_4$ .

*Proof.* The proof is similar to the proof of Theorem 3.4. ■

### 3.2.2 The Conjugacy Class Graph of Some Metabelian Groups of Order 16.

This section discusses the conjugacy class graphs of three metabelian groups of order 16 where the sizes of all conjugacy classes of metabelian groups of order 16 are used. We start with the dihedral group.

**Theorem 3.7.** Let  $G$  be a dihedral group of order 16,  $G \cong D_8 = \langle a, b \mid a^8 = b^2 = 1, bab = a^{-1} \rangle$ . Then, the conjugacy class graph of  $D_8$ ,  $\Gamma_{D_8}^{Cl} = K_5$ .

*Proof.* Based on Theorem 3.1, there are seven conjugacy classes in  $G$ , two of them are in the center of  $G$ . Thus the number of vertices in the conjugacy class graph,  $|V(\Gamma_{D_8}^{Cl})| = 7 - 2 = 5$ . Since two vertices are adjacent if their cardinalities are not coprime, thus  $\Gamma_{D_8}^{Cl}$  consists of one complete graph of  $K_5$  as illustrated in Figure 2.

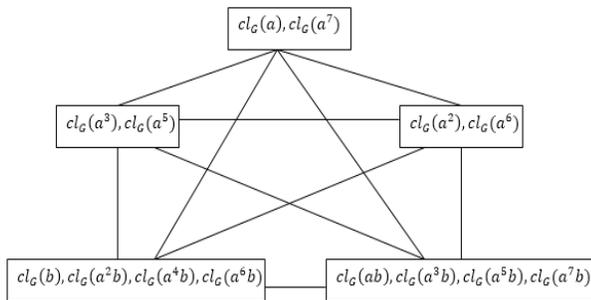


Figure 2 The conjugacy class graph of  $D_8$

From Theorem 3.7, the following corollary is obtained.

**Corollary 3.3** Let  $G$  be a dihedral group of order 16,  $G \cong D_8 = \langle a, b \mid a^8 = b^2 = 1, bab = a^{-1} \rangle$ . Then  $\chi(\Gamma_{D_8}^{Cl}) = \omega(\Gamma_{D_8}^{Cl}) = 5$  and  $\alpha(\Gamma_{D_8}^{Cl}) = \gamma(\Gamma_{D_8}^{Cl}) = 1$ .

*Proof.* The proof is clear since there is only one complete graph, namely  $K_5$ .

Next, the conjugacy class graph of quaternion groups of order 16 is presented.

**Theorem 3.8.** Let  $G$  be a quaternion group of order 16,  $G \cong Q_8 = \langle a, b \mid a^8 = 1, a^4 = b^2, aba = b \rangle$ . Then, the conjugacy class graph of  $Q_8$ ,  $\Gamma_{Q_8}^{Cl} = K_5$ .

*Proof.* Based on Theorem 3.2, there are five non-central conjugacy classes. Thus the number of vertices in the conjugacy class graph,  $|V(\Gamma_{Q_8}^{Cl})| = 5$ . Since two vertices are adjacent if their cardinalities are not coprime, thus  $\Gamma_{Q_8}^{Cl}$  consists of one complete graph of  $K_5$ , as claimed. ■

Based on Theorem 3.8, the following corollary is obtained.

**Corollary 3.4.** Let  $G$  be a quaternion group of order 16,  $G \cong Q_8 = \langle a, b \mid a^8 = 1, a^4 = b^2, aba = b \rangle$ . Then  $\chi(\Gamma_{Q_8}^{Cl}) = \omega(\Gamma_{Q_8}^{Cl}) = 5$  and  $\alpha(\Gamma_{Q_8}^{Cl}) = \gamma(\Gamma_{Q_8}^{Cl}) = 1$ .

**Theorem 3.9.** Let  $G$  be a Quasi-dihedral of order 16  $\cong Qh_{16} = \langle a, b \mid a^8 = b^2 = 1, bab = a^3 \rangle$ . Then, the conjugacy class graph of  $Qh_{16}$ ,  $\Gamma_{Qh_{16}}^{Cl} = K_5$ .

*Proof.* The proof is similar to the proof of Theorem 3.8. ■

## 4.0 CONCLUSION

In this paper, the conjugacy classes of some metabelian groups of order 16 were computed. The obtained results were then applied to graph theory, more precisely to the conjugate graph and conjugacy class graph of the groups. It is proven that the conjugate graph of all groups consists of five complete components, namely  $\bigcup_{i=1}^3 K_2 \cup \bigcup_{i=1}^2 K_4$ , while the conjugacy class graph of the groups in the scope of this paper consists only one complete graph of  $K_5$ . Besides, some graph properties are obtained for both graphs.

## Acknowledgement

The authors would also like to acknowledge Ministry of Higher Education (MOHE) Malaysia and UTM for the Research University Fund (GUP) for Vote No 08H07.

## References

- [1] Davidoff, G., Sarnak, P and ValeGe A. 2003. *Elementary Number Theory, Group Theory and Ramanujan Graphs*. 1st Edition. Cambridge: Cambridge University Press.
- [2] Rahman, S. F. A. and Sarmin, N. H. 2012. Metabelian Groups of Order At Most 24. *Menemui Matematik*. 34(1): 77-93.
- [3] Fraleigh, J. B. 2002. *A First Course in Abstract Algebra*. 7th Edition. New Jersey: Pearson.
- [4] Bondy, J. A. and Murty, U. S. R. 1982. *Graph Theory with Application*. 5th Edition. New York: Elsevier Science Publishing Co. Inc.
- [5] Godsil, C. and Royle, G. *Algebraic Graph Theory*. 5th Edition. London: Springer-Verlag.
- [6] Erfanian, A and Tolve, B. 2012. Conjugate Graphs of Finite Groups. *Discrete Mathematics, Algorithms and Applications*. 4(2):35-43.
- [7] Bertram, E. A., Herzog, M. and Mann, A. 1990. On a Graph Related to Conjugacy Classes of Groups. *Bulletin London Mathematical Society*. 22(6): 569-575.
- [8] Miller, G. A. 1944. Relative Number of Non-invariant Operators in a Group. *Proceeding of National Acaemic Science USA*. 30(2): 25-28.
- [9] Omer, S. M. S., Sarmin, N. H., Erfanian, A. and Moradipour, K. 2013. The Probability That an Element of a Group Fixes a Set and the Group Act on Set by Conjugation. *International Journal of Applied Mathematics and Statistics*. 32(2): 111-117.
- [10] Moreto, A., Qian, G and Shi, W. 2005. Finite Groups Whose Conjugacy Class Graphs Have Few Vertices. *Archiv der Mathematik*. 85(2): 101-107.
- [11] Bianchi, M., Herzog, M., Pacifici, E. and Saffirio, G. 2012. On the Regularity of a Graph Related to Conjugacy Classes of Groups. *European Journal of Combinatorics*. 33: 1402-1407.
- [12] Ilangovan, S and Sarmin, N. H. 2013. On Graphs Related to Conjugacy Classes of Some Two-groups. *AIP Conference Proceeding*. 1522: 872-874.
- [13] Moradipour, K., Sarmin, N. H. and Erfanian, A. 2013. On Graph Associated to Conjugacy Classes of Some Metacyclic 2-Groups. *Journal of Basic and Applied Scientific Research*. 3(1):898-902.