

## DESIGN AND ANALYSIS OF SUPER TWISTING SLIDING MODE CONTROL FOR MACHINE TOOLS

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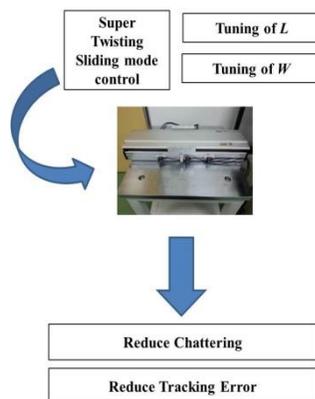
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### Graphical abstract



### Abstract

High demands of precision on machine tools are hardly cope by using existing classic control algorithms. This paper focuses on the design, analysis and validation of a super twisting sliding mode controller on a single axis direct drive positioning system for improved tracking performances. The second order positioning system parameters were determined using input and output of measured data. Effects of two gain parameters in control algorithm on the quality of the control input and tracking error were analysed experimentally. The gain parameters were selected based on magnitude reduction in chattering during practical application. The performance of tuned super twisting sliding mode controller was compared with a traditional sliding mode controller using sigmoid-like function. Results showed that super twisting sliding mode controller reduced the chattering effect and improved the performance of system in terms of tracking error by 16.5%.

Keywords: Accuracy, chattering, machine tools, super twisting, sliding mode control,

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## 1.0 INTRODUCTION

Over recent years, there has been an explosive growth of interest in the development of precision control especially in manufacturing sector on par with the sky-rocketing development of technology and demands. Many control algorithms had been proposed throughout the centuries to replace the traditional control approach in order to improve the accuracy and precision performance of machine tools. One of the popular approaches is the well-known nonlinear controller called, super twisting sliding mode controller (ST-SMC). ST-SMC provided similar robustness and disturbance rejection properties as sliding mode controller (SMC) but without the chattering effect [1]. The introduction of ST-SMC in literature is a great breakthrough as it suppressed the chattering effect, that is, the main drawback of traditional SMC without affected its performance [2].

The ST-SMC was proposed by Levant [3] and the general idea of ST-SMC is the enforcement of sliding modes with continuous control action to reduce chattering effect caused by discontinuous input [4]. This is because the chattering effect is mainly due to the high frequency oscillations as discontinuity is formed in signum function. The stability analysis and the finite convergence time also had been studied extensively in literature such as point-to-point method for state trajectories [5], Lyapunov function [6], and analysis in time domain [4].

The ST-SMC had been implemented in many fields of applications such as aerospace engineering [7], robot arms [8], diesel engines [9] and motors [10]. The twisting algorithm had been combined with ST-SMC to formed hybrid SMC in pitch control for aircraft in [7]. The ST-SMC was also applied to control the speed of diesel power generator and it is robust against speed and load change [9]. For motors applications, ST-SMC had been applied in switched reluctance motor for

speed control [10]. In addition, ST-SMC was combined with ST-SMC observer and tested with DC motor for the improvement of dynamic response and disturbance rejection [11]. Another interesting implementation of ST-SMC was in the output power control of wind energy conversion system based on permanent magnet synchronous generator [12].

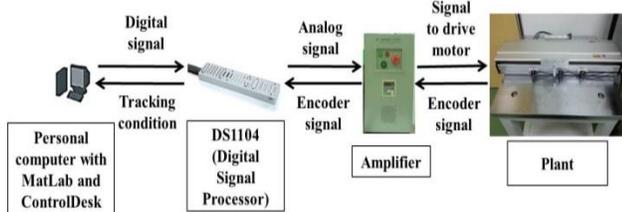
A survey on applications of second order sliding mode controller can be found in [13]. However, ST-SMC was not clearly recorded in that particular survey. In contrast, the ST-SMC observer in mechanical system was focus in [14]. Thus, there is still lacking of records of ST-SMC on the applications of mechanical system, specifically, machine tool applications.

The main objective of this paper is to justify and validate the performance and application of ST-SMC in machine tools. In addition, the analysis on the effect of parameters tuning for ST-SMC is also presented. The ST-SMC is applied on machine tools application, specifically, a single axis direct drive system with linear motor for positioning control. Section 2 describes the considered system as well as the experimental setup. Section 3 discusses the design of ST-SMC and the velocity estimator. In section 4, results and discussion on the analysis of the parameters tuning for ST-SMC are presented. Lastly, section 5 concludes and provides the future recommendation for the paper.

## 2.0 METHODOLOGY

### 2.1 Experimental Setup

The considered setup is a single axis direct drive system with an ironless flat linear motor. It is equipped with a 4  $\mu\text{m}$ -resolution of linear encoder. Figure 1 shows the configuration of the experimental setup. The single axis direct drive system is connected with the servo amplifier and linked with the dSPACE DS1104 data acquisition unit (DAQ). Then, the host computer equipped with MATLAB and ControlDesk softwares are connected to the DAQ.



**Figure 1** Configuration of the experimental setup

A single-input-single-output (SISO) model is used to describe the dynamics of the considered system and the frequency domain identification method is used to estimate the dynamics of system [15-16]. By using band-limited white noise excitation signal, the measured input voltage and output position signals are used to approximate the SISO frequency response

function (FRF) of the system through the H1 estimator [15-16]. The sampling frequency used is 5000 Hz. The nonlinear frequency domain identification method was used to fit a parametric model on the measured FRF [15-16]. A second order transfer function with time delay of 0.00045 seconds is shown in Equation (1) and it can be represented as Equation (2).

$$\frac{Y(s)}{U(s)} = \frac{A}{s(s+B)} \quad (1)$$

$$\ddot{y}(t) = -B\dot{y}(t) + Au(t) \quad (2)$$

where  $A = 7.5\text{e}8 \mu\text{m}/\text{V}\cdot\text{s}^2$ , and  $B = 3622 \text{ s}^{-1}$ ,  $U(s)$  or  $u(t)$  is the control input voltage to the drive in volt, V while  $Y(s)$  is the actual output position in the unit of micrometer. Both  $\dot{y}(t)$  and  $\ddot{y}(t)$  are the first and second time derivatives of actual output position.

### 2.2 Design of Super Twisting Sliding Mode Control

In order to design the ST-SMC, two components were considered, namely; switching function and control laws. The switching function used in this paper was the traditional SMC sliding surface,  $s(t)$  that related to the tracking error,  $e(t)$  and time derivative of tracking error,  $\dot{e}(t)$  [17] as shown in Equation (3) and (4).

$$s(t) = \left( \lambda + \frac{d}{dt} \right)^{n-1} e(t) \quad (3)$$

$$e(t) = y(t) - r(t) \quad (4)$$

where  $\lambda$  is a positive constant while  $n$  is the order of uncontrolled system.  $r(t)$  and  $y(t)$  represents the desired position and actual output position respectively. Equations (5) and (6) show the first time derivative of sliding surface,  $\dot{s}(t)$  and second time derivative of tracking error,  $\ddot{e}(t)$  respectively. Equation (7) was formed through substitution of Equation. (2) and (6) into Equation (5).

$$\dot{s}(t) = \lambda\dot{e}(t) + \ddot{e}(t) \quad (5)$$

$$\ddot{e}(t) = \ddot{y}(t) - \ddot{r}(t) \quad (6)$$

$$\dot{s}(t) = \lambda\dot{e}(t) + Au(t) - B\dot{y}(t) - \ddot{r}(t) \quad (7)$$

Based on Equation (7), the equivalent control,  $u_{eq}$  was obtained when  $\dot{s}(t) = 0$ . The control laws of ST-SMC are stated in Equations (8) and (9).

$$u(t) = u_{eq} - L|s(t)|^{0.5} \text{sign}(s) + u_1(t) \quad (8)$$

$$\dot{u}_1(t) = -W \cdot \text{sign}(s) \quad (9)$$

where both  $L$  and  $W$  are positive constant while  $u_1(t)$  is the original discontinuous input from traditional SMC [4]. Figure 2 shows the general block diagram of the designed ST-SMC.

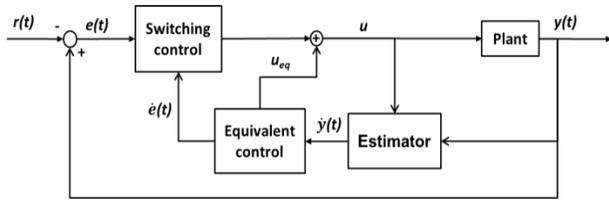


Figure 2 General block diagram of ST-SMC

### 3.0 RESULTS AND DISCUSSION

Based on Figure 2, the experiment was conducted. A desired sinusoidal reference input,  $r(t)$  with amplitude of 20 000  $\mu\text{m}$  and frequency of 0.5 Hz was used to excite the system for tracking control. The signals of control input,  $u(t)$  and tracking error,  $e(t)$  were measured and captured. A Kalman-Bucy filter was designed and used to estimate the velocity of the system to reduce the noises [18]. The  $\lambda$  of  $s(t)$  was set as 800 for all tests.

#### 3.1 Analysis on Effect of Parameter $L$

Parameter  $L$  was varied to analyse its effect in terms of control input and tracking error. The initial value of parameter  $L$  was set as  $1 \times 10^{-5}$  (datum). The parameter  $W$  was set as constant for the analysis on effect of parameter  $L$ . A total of 4 tests were conducted where the parameter  $L$  is set as  $1 \times 10^{-5}$ ,  $2 \times 10^{-5}$ ,  $4 \times 10^{-5}$ , and  $6 \times 10^{-5}$  respectively. The root mean square error (RMSE) was used to compare the effect of changing parameter  $L$  on tracking error as shown in Table 1. Figures 3 and 4 show the control input and tracking error of the test with vary of parameter  $L$  respectively.

Table 1 RMSE of changing parameter  $L$

Parameter $L(x 10^{-5})$	RMSE (micron)
1	28.1761
2	8.5434
4	2.2898
6	2.1933

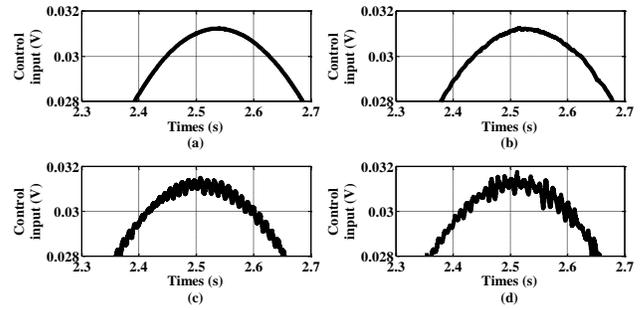


Figure 3 Control input obtained when parameter of  $L$  is (a)  $1 \times 10^{-5}$ , (b)  $2 \times 10^{-5}$ , (c)  $4 \times 10^{-5}$  and (d)  $6 \times 10^{-5}$  respectively

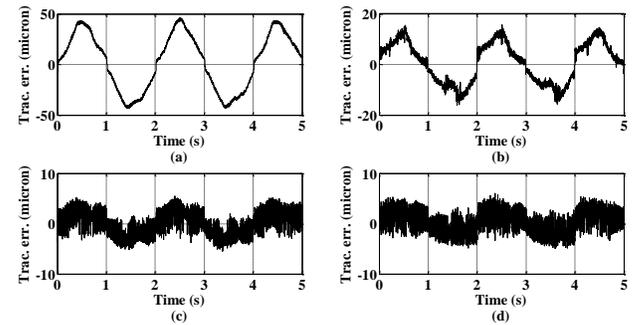
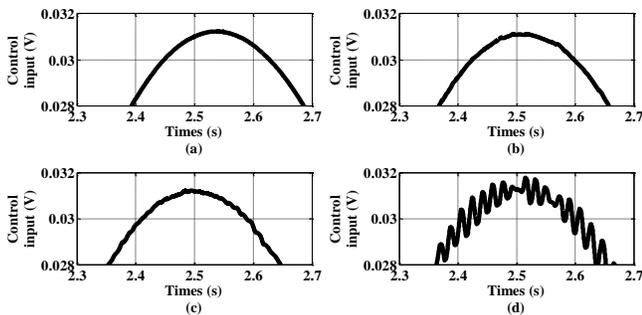


Figure 4 Tracking error obtained when parameter  $L$  is (a)  $1 \times 10^{-5}$ , (b)  $2 \times 10^{-5}$ , (c)  $4 \times 10^{-5}$  and (d)  $6 \times 10^{-5}$  respectively

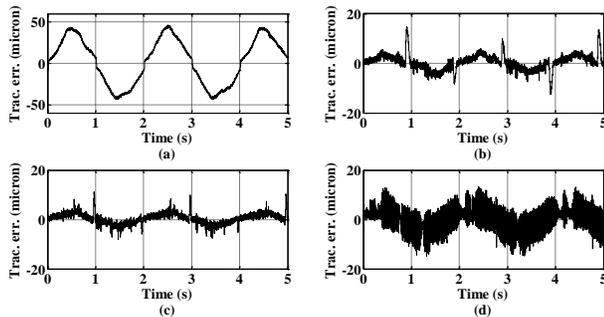
Based on Figure 3, the increment of parameter  $L$  caused the control input signal became noisy. It could be viewed that the noises had polluted the control input especially on the use of large  $L$ . The noises (chattering) became worsen when the parameter  $L$  was increased. In contrast, the tracking error was reduced when the parameter  $L$  was increased (refer to Figure 4). The RMSE results in Table 1 showed that although the increment of parameter  $L$  reduced the tracking error, the degree of improvement was decreased. When the parameter  $L$  was increased from  $1 \times 10^{-5}$  to  $2 \times 10^{-5}$ , the percentage of improvement for tracking error is around 69.7%. However, the percentage of improvement was only 4.2% when the parameter  $L$  was increased from  $4 \times 10^{-5}$  to  $6 \times 10^{-5}$ . As a summary, the increment of parameter  $L$  improved the tracking error but more noises were found in control input, specifically, increased the chattering. This phenomenon is aligned with theory as increment of gain value decreased the tracking error but will excite the high frequency oscillations [19]. In addition, the degree of improvement was not constant. Based on this analysis, the parameter  $L$  chosen was  $4 \times 10^{-5}$  as it provided a large improvement of tracking error (91.9%) compared to the initial value. It is not worth to use  $6 \times 10^{-5}$  as the tradeoff of chattering is too high compared to the low percentage of improvement.

**3.2 Analysis on Effect of Parameter W**

A total of 4 tests were conducted by varying the parameter  $W$  to analyse its effect in terms of control input and tracking error on the positioning system. The initial value (datum) of  $W$  was set as  $1 \times 10^{-5}$  (to align with analysis on parameter  $L$ ) while the other 3 tests were using  $2 \times 10^{-2}$ ,  $5 \times 10^{-2}$ , and  $5 \times 10^{-1}$  respectively as the value of  $W$ . The value of  $5 \times 10^{-1}$  was used to show the impact of large  $W$  towards tracking error and chattering. The parameter  $L$  was set as constant. Similarly to section 4.1, the RMSE was used to examine the effect of varying parameter  $W$  towards tracking error. Table 2 shows the RMSE results of all 4 tests while Figure 5 and 6 show the control input and tracking error of the tests graphically.



**Figure 5** Control input obtained when parameter  $W$  is (a)  $1 \times 10^{-5}$ , (b)  $2 \times 10^{-2}$ , (c)  $5 \times 10^{-2}$ , and (d)  $5 \times 10^{-1}$  respectively



**Figure 6** Tracking error obtained when parameter  $W$  is a)  $1 \times 10^{-5}$ , (b)  $2 \times 10^{-2}$ , (c)  $5 \times 10^{-2}$ , and (d)  $5 \times 10^{-1}$  respectively

**Table 2** RMSE of changing parameter  $W$

Parameter $W$	RMSE (micron)
$1 \times 10^{-5}$	28.1761
$2 \times 10^{-2}$	3.0351
$5 \times 10^{-2}$	2.2749
$5 \times 10^{-1}$	5.1564

According to Figure 5, the increment of  $W$  eventually increased the noises in control input (especially on Figure 5d). The chattering occurred

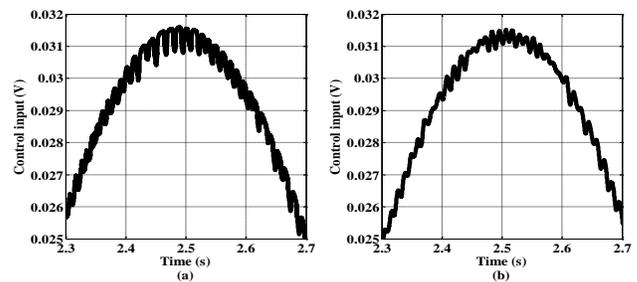
when the value of  $W$  is considerable large. On the other hand, the tracking error was decreased when  $W$  was increased (refer to Figure 6). The RMSE results in Table 2 also showed the decrement of tracking error except the test where  $W = 5 \times 10^{-1}$ . This was due to the occurred chattering and thus, affected the performance of the control algorithm (refer to Figure 6d). Furthermore, spikes were formed with the increment of  $W$  and it affected the overall RMSE results. As a summary, the tracking error was reduced with the increment of  $W$ . However, the tradeoff of improved tracking error was the chattering and the formation of spikes. Based on this analysis, the value of  $W$  chosen is  $5 \times 10^{-2}$  as it provided the largest degree of improvement in terms of tracking error (91.9%) and only small spikes were formed. No significant chattering was observed for the chosen  $W$  and its variation is desirable in the region of  $10^{-2}$ .

**3.3 Performance of ST-SMC**

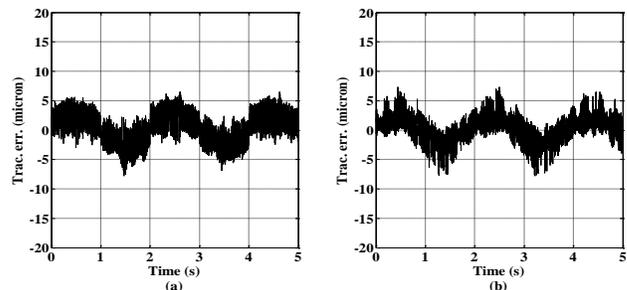
A traditional SMC with sigmoid-like function was designed and was compared with the designed ST-SMC in terms of control input and tracking error. Same  $\lambda$  value and sliding surface (Equation (3) and (4)) was used for the SMC. By using  $L = 4 \times 10^{-5}$  and  $W = 5 \times 10^{-2}$  for ST-SMC, the comparison results were showed in Table 3, Figure 7 and 8.

**Table 3** RMSE for SMC and ST-SMC

Controller	RMSE (micron)
SMC	2.6077
ST-SMC	2.1784



**Figure 7** Control input for (a) SMC and (b) ST-SMC



**Figure 8** Tracking error for (a) SMC and (b) ST-SMC

Based on Figure 7 and 8, the noises in control input and tracking error of ST-SMC was lesser compared to SMC as the control input signal for ST-SMC was smoother. It showed that the chattering was reduced by using parameters chosen for the designed ST-SMC compared to SMC. Although sigmoid function was applied to reduce chattering in SMC, the chattering reduction ability of ST-SMC is still more pronounced. The ST-SMC also provided lower RMSE value compared to SMC which showed that ST-SMC improved the tracking overall performance (refer to Table 3). This is due to the fact that the smoothing of control input reduced the chattering and resulted in reduction of tracking error. By utilizing the analysis in section 4.1 and 4.2, the chosen parameters values are mainly focused in reducing the chattering (noises) as well as tracking error for practical application.

#### 4.0 CONCLUSION AND SUGGESTIONS

The ST-SMC was designed and applied on a single axis positioning system to improve the tracking performance of the system. The analysis of the effect on parameters  $L$  and  $W$  provided information for the value selection. Based on the analysis, the large value of  $L$  produced low tracking error but caused the chattering to occur. The large value of  $W$  also incurred chattering and formation of spikes although reduced the tracking error. The variation of  $L$  is minimal which is still in the region of  $10^{-5}$  while  $W$  provided larger variation which is in the region of  $10^{-2}$  for better chattering reduction. Tradeoff between chattering and tracking error needed to be considered based on the demand. In this case, the reduction of chattering is preferred for practical application. For future recommendation, a more comprehensive analysis of performance could be performed to identify the chattering frequencies. The dynamic friction models also could be added in order to compensate the friction. Artificial intelligent also could be utilized to counter the spikes issue in tracking for better accuracy.

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