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A NUMERICALLY CONSISTENT MULTIPHASE POISEUILLE FLOW COMPUTATION BY A NEW PARTICLE METHOD

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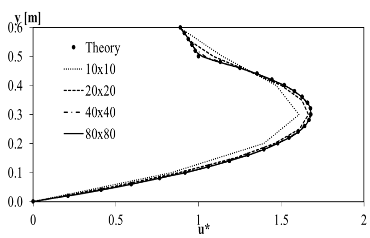
Graphical abstract



Abstract

Recently, there is a rising interest in simulating fluid flow by using particle methods, which are mesh-free. However, the viscous stresses (or diffusion term) appeared in fluid flow governing equations are commonly expressed as the second-order derivatives of flow velocities, which are usually discretized by an inconsistent numerical approach in a particle-based method. In this work, a consistent method in discretizing the diffusion term is implemented in our particle-based fluid flow solver (namely the Moving Particle Pressure Mesh (MPPM) method). The new solver is then used to solve a multiphase Poiseuille flow problem. The error is decreasing while the grid is refined, showing the consistency of our current numerical implementation.

Keywords: Poiseuille flow, particle method, Moving Particle Semi-implicit (MPS), Moving Particle Pressure Mesh (MPPM), multiphase flow, CFD

1.0 INTRODUCTION

Recently, the particle method such as the Moving Particle Semi-implicit (MPS) has featured its remarkable flexibilities in handling interface deformations as well as fragmentations (Koshizuka *et al*. [1,2]). In the framework of particle method, the fluid governing equation is applied on individual moving particles, thereby avoiding the spatial discretization of the convective term which is prone to numerical instability.

However, in order to ensure the workability of MPS method, most of the current works in MPS require parameter-tuning and artificial numerical treatments to ensure a successful fluid flow simulation. For example, the widely used minimum pressure gradient model contains an artificial repulsive force term to ensure numerical stability [1]. Besides that, the collision model is normally employed to modify the velocity field, which involves a proper tuning of collision ratio (denoted as ε in [3,4]). Furthermore, tuning is necessary in order to reach a compromise between volume conservation as well as smoothness of pressure field [5-7].Very recently, we have addressed these problems and proposed a new particle method to circumvent the above problems [8].

Another existing problem in MPS is the inconsistency of its Laplacian operator used to discretize the viscous (or diffusion) term of fluid flow [9]. In [9], we have shown mathematically that the MPS Laplacian operator is inconsistent (i.e. error is increasing as the grid is refined) when irregular grid is encountered. Recently, by making use of the Taylor series, Luo *et al*. [10] have developed the Consistent Particle Method (CPM) to represent the derivatives appeared in the Navier-Stokes equations in a numerically consistent manner. In this paper, the

CPM is implemented in the framework of MPPM and its application in simulating the multiphase Poiseuille flow will be shown.

2.0 GOVERNING EQUATIONS

The two-dimensional multiphase, incompressible and isothermal Poiseuille flow is considered in the current work, whereby the flow field can be expressed by the mass conservation (continuity) equation:

$$\nabla \bullet \vec{u} = 0 \quad \text{(Eq. 1)}$$

and the momentum conservation equations:

$$\rho \frac{Du}{Dt} = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + S_x \quad \text{(Eq. 2)}$$

$$\rho \frac{Dv}{Dt} = -\frac{\partial P}{\partial y} + \frac{\partial}{\partial x}\left(\mu \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial y}\right) \quad \text{(Eq. 3)}$$

In the above equations, ρ is fluid density, P is fluid pressure, μ is fluid dynamic viscosity, S_α is the source term (e.g. driving force) in the α-direction and $\vec{u}$ = <u,v> is the velocity vector. It is important to note that the fluid properties such as μ are no longer constant throughout the computational domain. The above equations can be solved by our MPPM approach as reported in our earlier work [8]. In the current paper, we intend to seek for a consistent method to discretise the viscous terms appeared in the momentum conservation equations as described below.

2.1 Consistent Particle Method

Recently, Luo *et al*. [10] have proposed their CPM method in discretizing the differential operators appeared in the above governing equations. In MPPM, since the pressure is stored at the Eulerian mesh, the Poisson equation can be readily discretized by the standard finite volume method. However, moving particles (in the numerical framework of MPPM) which are carrying velocity information are scattered within the flow domain [8]. Therefore, the development of a numerical technique which is capable to accurately discretize the derivative terms based on randomly scattered data points is of vital importance. The detailed algorithm of MPPM can be found in [8].

The idea of CPM is basically stemming from the finite difference and the least square methods. And, by realizing that the shear stress is constant near the fluid interface, Luo *et al*. [10] have shown mathematically that the viscous term (e.g. x-momentum equation) can be discretized as:

$$\frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = \sum_{j \neq i} \mu_{ij} C_j (u_j^n - u_i^n) \quad \text{(Eq. 4)}$$

whereby the harmonic mean form of the interacting viscosity μ_{ij} must be pursued:

$$\mu_{ij} = \frac{2\mu_i \mu_j}{\mu_i + \mu_j} \quad \text{(Eq. 5)}$$

Here, C_j is the coefficient, which is obtained via the inversion of a 5x5 matrix (for 2D) [10].

3.0 RESULTS AND DISCUSSION

3.1 Poiseuille Flow

A multiphase Poiseuille flow case is computed to examine the capability of MPPM-CPM in handling multi-viscosity fluid flow system. Figure 1 illustrates the schematic diagram of the flow case, in which the fluids are contained within two stationary infinite top and bottom stationary walls. The flows are driven by a pressure gradient S_x = -0.5Pa / l.

Here, the infinite walls are modelled via implementing a periodic boundary condition at both the inflow (left edge) and outflow boundaries (right edge). In the current flow case, the length of the wall l is prescribed as l =1.0m, whereby the thicknesses and densities of both fluid layers are similar to each other, i.e. d_1=d_2=0.5m and $\rho_1 = \rho_2$ = 2kg/m^3. The lower fluid (i.e. Fluid 1) has a dynamic viscosity μ_1=0.05Pa.s while the dynamic viscosity of the top fluid (Fluid 2) can be calculated as μ_2= μ_1/M, where M =1/8. For this problem, the theoretical solution is available and it can be found from the earlier MPS work [3]. The dimensionless x-velocity can be determined as u^* = u/U_0, where U_0 is the interfacial velocity.

Figure 2 shows the flow velocity along the y-direction at station x=0.5m. The discontinuity in the velocity derivative can be clearly seen at the fluid interface due to the viscosity jump. This discontinuity can be better resolved upon grid refinement (see the convergence of velocity profiles towards the theoretical solution at y=0.5m in Figure 2). Figure 3 reports quantitatively the solution errors obtained by the current MPPM-CPM approach. Upon the implementation of CPM, the error is reduced progressively upon the grid refinement, showing the consistency of the current approach. The order of accuracy of the MPPM-CPM approach is hovering around 1.32-1.75 (see Table 1), while the solutions of the MPPM-MPS approach show lack of consistency (order of accuracy ~0) at grid level 80x80.

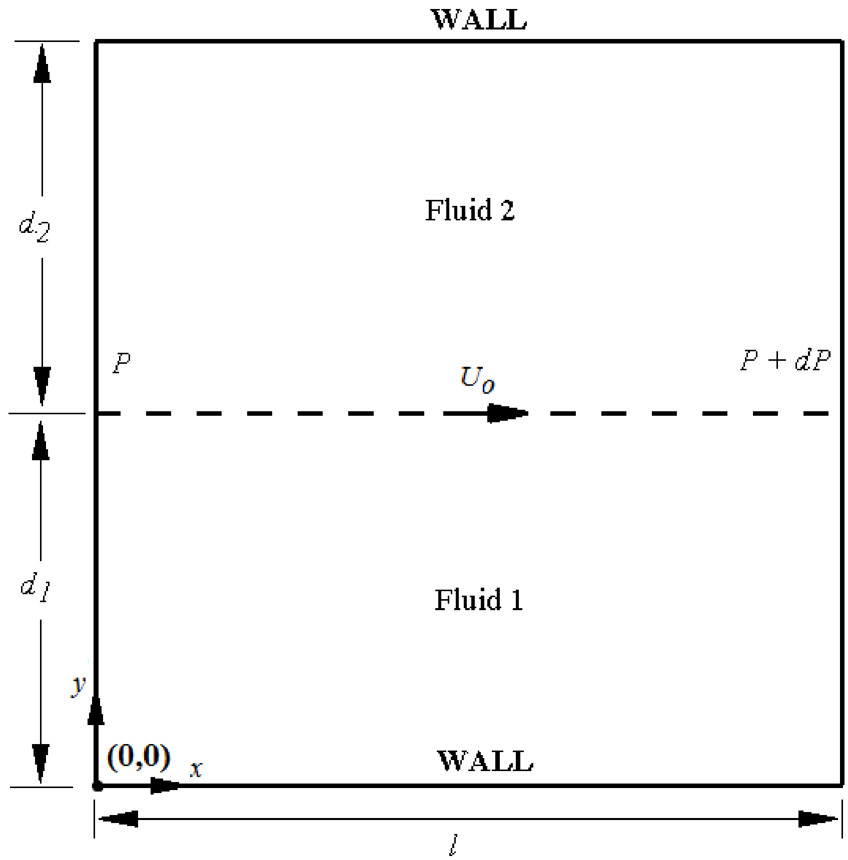


Figure 1 Schematic diagram of two-phase Poiseuille flow between two parallel walls

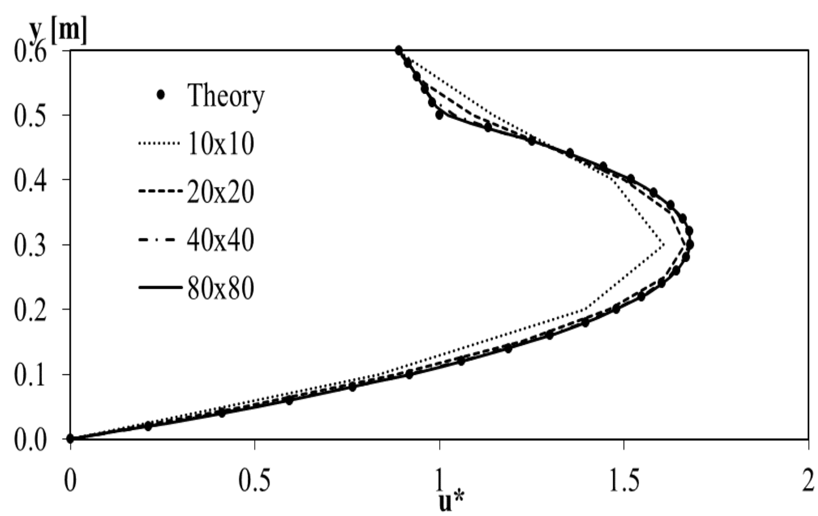


Figure 2 Velocity profiles at section x=0.5m predicted by MPPM-CPM approach

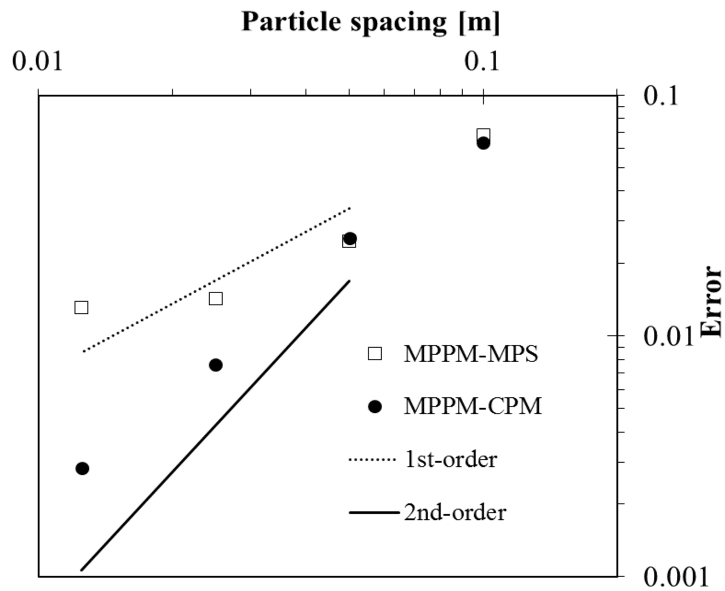


Figure 3 Numerical errors at various particle spacing

Table 1 Order of accuracy of different numerical approach

Mesh	Spacing [m]	MPPM-MPS		MPPM-CPM	
		Error	Order	Error	Order
10x10	0.1	0.06810	-	0.06388	
20x20	0.05	0.02468	1.465	0.02554	1.32
40x40	0.025	0.01427	0.790	0.00759	1.75
80x80	0.0125	0.01313	0.120	0.00284	1.42

4.0 CONCLUSION

Numerical inconsistency has been found as the MPS scheme (i.e. MPPM-MPS) is used to discretize the viscous term appeared in the flow governing equation. This issue can be circumvented by the recently proposed Consistent Particle Method (CPM), whereby the numerical consistency is assured via the Taylor series. The CPM method has been implemented in our MPPM solver to simulate multiphase flow. As seen, the results are numerically consistent and the order of accuracy is > 1. The extension of the MPPM-CPM approach to handle the more complex multiphase flow cases is underway.

Acknowledgement

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