

# Autonomous Attitude Control of a Quadcopter Unmanned Aerial Vehicle (UAV)

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**Abstract**— This paper discusses on the development of autonomous attitude control of a quadcopter unmanned aerial vehicle (UAV). The Quadcopter system is modeled using state-space equations and simulated in MATLAB/SIMULINK. The paper describes the controller design method for automatic attitude control of UAV to enable the system to operate based on the task of the navigation unit. To be specific, the PID controller is designed to control and stabilize quadcopter attitude. Then Quadcopter acceleration control system will be designed. This system is designed by combining the attitude control on Quadcopter rigid body with acceleration control Quadcopter movement. Through the obtained results, the proposed PID controller managed to control the acceleration of quadcopter well.

**Index Terms**—About four key words or phrases in alphabetical order, separated by commas.

## I. INTRODUCTION

The Unmanned Aerial Vehicle (UAV) has gained demand on the worldwide markets. There are two types of UAV which is fixed wing and rotary wings. Nowadays, one of the most popular rotary wings UAV is quadcopter. It has more advantages in term of their capability to take off and landing in vertical condition. In addition, it has good mobility, simple mechanics, and ability on load capacity [1]-[3].

The studies on quadcopter UAV modeling and control have been increasing rapidly, recently. A number of examples of these studies can be summarized as follows; A. Bourbon modeled using the force moment balance and simulate a small quadcopter model using Matlab/Simulink [4]. Bora Erginer et al. [5] also has done the quadcopter modeling and simulating PD control of a quadcopter using vision to estimate the relative position. Meanwhile, Tommaso Bresciani modeled a quadcopter using Newton -Euler method and PID control algorithms was proposed and the proposed method was analyzed in 3D graphic simulation [6].

Nowadays, development of autonomous UAV has been growing fast as it is more reliable in applications. The system can be categorized into three types, i.e. automatic landing system, automatically take off system, and the automatic navigation system.

An automatic landing system requires landing the quadcopter safely. Safe landing is much more critical because of the uncontrolled in decreasing the motor speed will

significantly affect the body of quadcopter to drop drastically and leads to unsafe landing.

The automatic take-off systems used to initiate the UAV to fly from the ground autonomously. The system should ensure the height of quadcopter in a consistent condition. In this case, each of four motor propeller must produce a force against the gravitational force to enable them to lift the quadcopter. The good performance of the automatic vertical takeoff system indicates how fast the quadcopter achieves its desired altitude in steady state. The performance of the system also signifies how smooth it maintains the height of quadcopter.

In regards to hovering control of quadcopter UAV, several results can be found in previous research work. For example, in [8], the fuzzy PID had been proposed to adjust the PID gains automatically when the prescribed altitude is changed. However, the proposed method cannot be possible to be implemented practically as the method assumed zero delay every time the PID is tuned. Meanwhile, [9] presented the PID control for altitude control experimentally. The xPC Target is used to link between Matlab/Simulink and quadcopter. The PID controller is tuned as  $K_p=0.06$ ,  $K_i=0.5$ ,  $K_d=1.6$  with the payload of quadcopter is 2kg, the system takes 7second to reach a 30cm reference altitude.

Besides that, the attitude of quadcopter controller also should be designed. The control system is designed to produce the desired acceleration of the attitude control autonomously. Both angle of  $\theta$  and  $\phi$  would produce torque to generate movement based on the E-frame where each movement about straightway fly and sideways fly. The acceleration controller of quadcopter movement is a major part in the development an autonomous Quadcopter. The controller used to ensure the movement is corresponds with the specified algorithm task. Hence, this paper describes a method to design Quadcopter attitude control system that consists of the rolling, pitching and yawing control. The attitude controller of quadcopter is about to control the angle of quadcopter body frame to generate the movement acceleration. The control system is designed to produce the desired acceleration by controlling the attitude autonomously. Both angles of rolling and yawing would produce torque to generate movement based on the E-frame where each movement about straightway fly and sideways fly. The acceleration controller of quadcopter movement is a major part in the development an autonomous Quadcopter. The controller ensures the movement is corresponded with the specified algorithm task from the

navigator unit.

Inkyu Sa and Peter Corke [7] provided the proof of quadcopter identity, valuation and control of translational gesture and heading angle using the open source quadcopter in details by using the MikroKopter. Computationally low-cost multi-amount velocity estimator was designed based on the changing aspects of the integrated measuring device, roll and pitch attitude controller, and system latencies. All data fuses from the integrated inertial sensors and a low-rate on-board laser finder. Nested loop structure is used to perform the controller. The outcome of estimator and closed-loop positioning test was compared to the ground truth analysis of the motion capture system. However, these approaches become more complicated by involving a lot of sensor in the system. In this paper, a new approach acceleration controller design was described by implementing of angular controller into the acceleration controller. This will be reduce the type of sensor that is required to be used in the system.

Quadcopter mathematical model will be used in developing the control system. Equation 1 shows the mathematical model used in this paper. The result parameters of the work were used in this paper for designing the attitude controller and it is shown in Equation (1)[6].

$$\begin{bmatrix} \ddot{X} = (\sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi) \frac{U_1}{m} \\ \ddot{Y} = (-\cos \psi \sin \phi + \sin \psi \sin \theta \cos \phi) \frac{U_1}{m} \\ \ddot{Z} = -g + (\cos \phi \cos \theta) \frac{U_1}{m} \\ \dot{p} = \frac{U_2}{I_x} \\ \dot{q} = \frac{U_3}{I_y} \\ \dot{r} = \frac{U_4}{I_z} \end{bmatrix} \quad (1)$$

Where  $\ddot{X}$  is sideway fly acceleration,  $\ddot{Y}$  is f straightway fly acceleration,  $\ddot{Z}$  is the lift up and downward acceleration,  $\dot{p}$  is the angular velocity of rolling,  $\dot{q}$  is the angular velocity of pitching,  $\dot{r}$  is the angular velocity of yawing. While the input  $U_1, U_2, U_3,$  and  $U_4$  can be summarized as follows:

$$\begin{bmatrix} U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 = bl(-\Omega_2^2 + \Omega_4^2) \\ U_3 = bl(-\Omega_1^2 + \Omega_3^2) \\ U_4 = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \end{bmatrix} \quad (2)$$

After understand the quadcopter model equations, its practical parameters have to be identified. The parameters are; the acceleration of gravity ( $g$ ); mass ( $m$ ) of quadcopter body, force lift factor ( $b$ ), center to end of cross body ( $l$ ), speed of motor ( $\Omega$ ) and; inertial ( $I$ ) parameters. This physical parameter will be then adopted in the simulation later.

Structure of this paper is as follows: In section II, the Quadcopter physical parameter is described. The propose of an attitude control of quadcopter controller is explained in section III and quadcopter acceleration control is given section IV. Conclusion is then drawn in Section V.

## II. QUADCOPTER PHYSICAL PARAMETER IDENTIFICATION

### A. Motor and propeller model

Lift factor can be determined using two different approaches, which is either by the blade element theory of force lift test experiment or the blade element theory calculation. In this paper, force lift test method is chosen because the blade element analysis requires a lot of variables, such as the environmental element and the propeller blade specification and these are hard to be obtained. On top of that, force lift test experiment deals with real time measurement; hence the parameter can be much more reliable. In this part, Electronic speed controller is used and triggered by PWM generator. The motor propeller speed is measured by using the micro tachometer. The lift force was measured using the weight balancer. The experiment analysis for this model is shown in Figure 1.

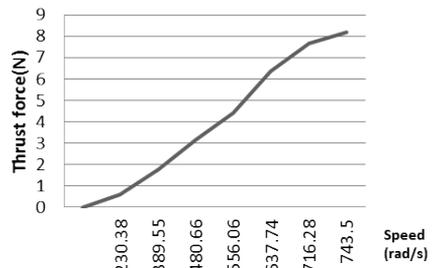


Figure 1: Force VS speed of propeller

Figure 1 shows the relationship between thrust forces produced by propeller speed. Theoretical aerodynamic has proven that thrust force varies proportionally with squaring velocity. To define the constant values of the relationship, the equation (3) is applied.

$$b = \frac{f}{\Omega_m^2} \quad \text{Thus} \quad b = 1.50 \times 10^{-5} \quad (3)$$

According to aerodynamic theory, drag factor depends on lift factor by a ratio of  $d/b = \tan \alpha$ , where  $\alpha$  is attack angle of propeller [13].

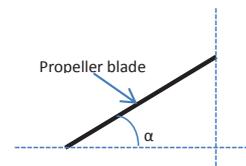


Figure 2: Attack angle of propeller

The value of  $\alpha$  equal to  $15^\circ$ , and drag factor is equal to  $d=4.02 \times 10^{-6}$ .

### B. Movement of inertia

The mechanical structure of the quadcopter that is developed in this work is made from aluminum. All

components are assembled to the mechanical structure. The top view of the mechanical structure can be seen in Figure 3.



Figure 3: Cross structure of the developed quadcopter frame

The chassis is made of aluminum profiles. All the components are assembled to the chassis. To assemble the components, screws are used. In the previous part, inertia matrix is involved in the quadcopter modeling. Hence, movement of inertia has been determined for this structure. Movement of inertia is about the dynamic characteristic of the rotational Quadcopter system on the B-frame. The general inertia matrix expresses as follows:

$$I = \begin{bmatrix} I_{XX} & -I_{XY} & -I_{XZ} \\ -I_{YX} & I_{YY} & -I_{YZ} \\ -I_{ZX} & -I_{ZY} & I_{ZZ} \end{bmatrix} \quad (4)$$

Since the Quadcopter frame designed with symmetrical structure, the inertia matrix can be assumed to be diagonal. Hence, the value of  $I_{XY}$ ,  $I_{XZ}$ ,  $I_{YX}$ ,  $I_{YZ}$ ,  $I_{ZX}$ , and  $I_{ZY}$  in the inertia matrix are assumed to be zero. The value of  $I_{XX}$ ,  $I_{YY}$ , and  $I_{ZZ}$  as shown in Figure 4 will be determined in this section.

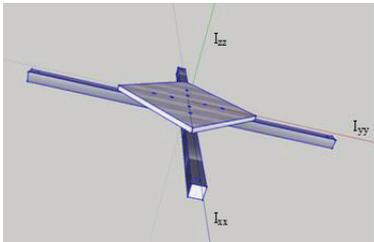


Figure 4: Three moments of inertia

The rotational movement of the quadcopter system is defined by three trajectories, which is rolling, pitching and yawing. The moment inertia parameter can be determined by using two approaches, which is by calculation and bifilar pendulum experiment. The bifilar pendulum have been done and the value of inertia for this model is shown in Table 1.

Table 1  
Moment of inertia value

Moment of inertia	Period per cycle (s)	Value (kgm <sup>2</sup> )
$I_x$	1.40	0.0398
$I_y$	1.40	0.0398
$I_z$	1.85	0.0421

### III. ATTITUDE OF QUADCOPTER

The movement concept of the X direction is similar to the Y direction movement concept, thus, only one movement will be described in this paper, which is the rolling control.

#### A. Rolling Control

The rolling control system is designed to produce a rolling movement at the target rolling angle. This rolling movement rate would generate a force to move to the right or to the left trajectory based on the E-frame. The control system constructed based on the simplification of mathematical model. The translational angular velocity of the quadcopter in x-axis can be obtained by using the following equation.

$$\dot{p} = \frac{U_2}{I_x} \quad (5)$$

where

$$U_2 = bl(-\Omega_2^2 + \Omega_4^2) \quad (6)$$

The rolling torque was generated by manipulating the changes of motor speed for motor 2 and opposite changes for motor 4. To simplify the simulation, the rate of changes on motor 2 is equal to the rate of change of motor 4. The angle of the rolling must have the cut off rate to avoid the quadcopter system overturn and may cause the system to collapse and also to ensure the generated torque is suitable for the rolling control system. Thus, the translation angle rate was limited to 30° from total rolling angle. Figure 5 shows the Simulink block of the rolling control system with PID.

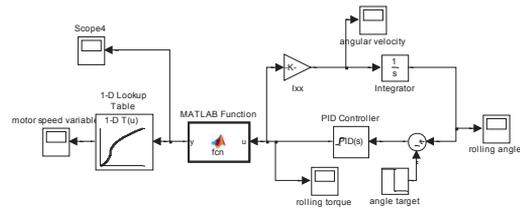


Figure 5: Simulink block of the rolling control system with PID.

In this part, the roll angle rate target is set to the maximum of 30° of rolling angle. The PID controller was tuned and the value of Controller parameters for PID; proportional = -0.1323, integral = -0.00351, derivative = 0.02 and filter coefficient = 6.411. Based on the result, the control system required less than 3s to reach 30° rolling angle target. The result is shown in Figure 6.

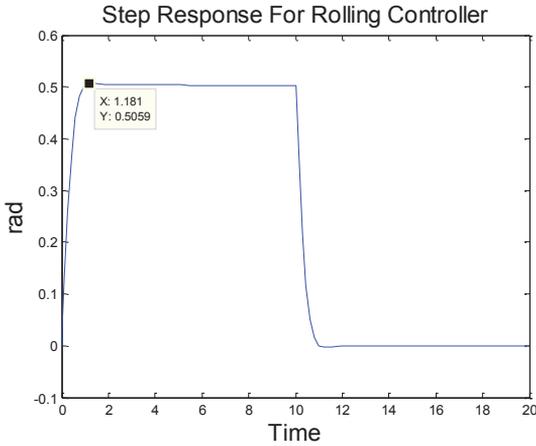


Figure 6: Rolling controller step response for 30° angle target.

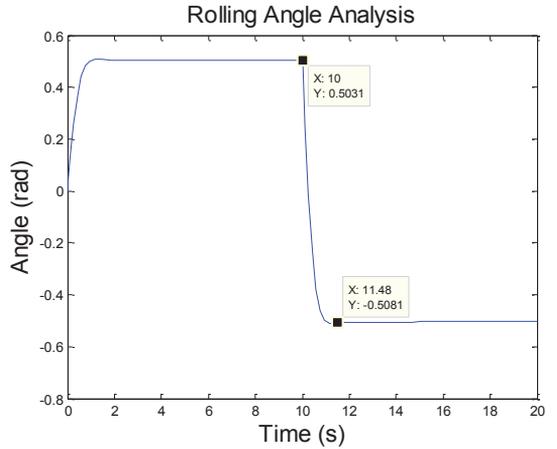


Figure 8: Rolling controller step response for 30° to -30° angle target

In terms of the speed of the propeller, the speed of both motor propellers ( $-\Omega_2 + \Omega_4$ ) initiated with the 100rad/s changes. Then the change drops to -7.525rad/s at 1 second, before the system reach target angle of 30° of rolling at 3.2 second. The speed of propeller response recorded as shown in Figure 7.

Based on the result, the control system only takes 1.48 second to change the rolling rate from 30° to the -30°. Besides that, the motor speed variable also should be concerned in the analysis. The analysis used to observe the capability of the actuator to receive an input signal regarding on the controller system requirement. The motor speed variable response as shown in Figure 9. Based on the response, maximum speed of change is 169.5rad/s. If the initial condition of the system maintains the altitude with the 400rad / s speed of the motor, it means the maximum motor speed of the system is 569.5rad/s, and it's still under the breakage speed of the motor.

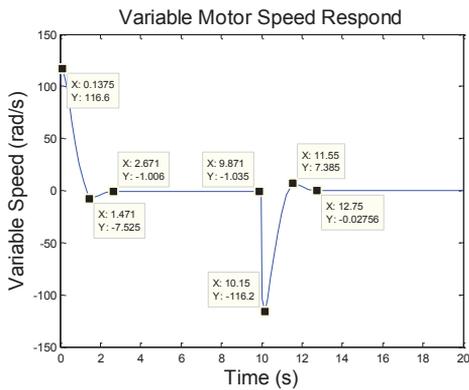


Figure 7: Rate of motor speed variable.

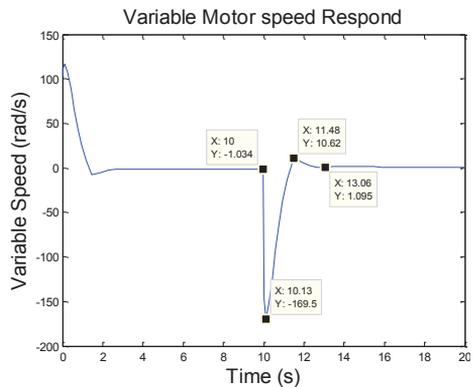


Figure 9: Variable changes motor speed.

In this paper, the control system is also tested for the extreme rolling rate changes. This is required to analyze the reliability of controller response at large rolling rate changes. The control system has been simulated to observe the response when the roll angle rate changes from +30° to the -30°. The result shows that the control system is reliable for aggressive changes of rolling rate. The response is as shown in Figure 8.

**B. Yawing Control**

The control system is designed to produce a yawing movement at the rate required. This yawing movement rate would generate torque to rotate and change orientation of the rigid body position either in clockwise or anticlockwise. The control system constructed based on the simplification of mathematical model that has been derived. The translational angular velocity of the quadcopter in x-axis can be obtained

by using the following formula.

$$\dot{r} = \frac{U_4}{I_z} \quad (7)$$

where

$$U_4 = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \quad (8)$$

The equation shows the input torque to generate yaw movement is to change the speed of the two motors that rotate clockwise ( $\Omega_1, \Omega_3$ ) in either increased or decreased, at the same time change the speed of the two motors that rotate counterclockwise ( $\Omega_2, \Omega_4$ ) at odds with the speed change motor that rotates clockwise. To simplify the simulation, the rate changes on motor 2 and motor 4 is equal to the rate of changing on motor 1 and motor 3. Figure 10 shows the Simulink block for the rolling control system with PID controller.

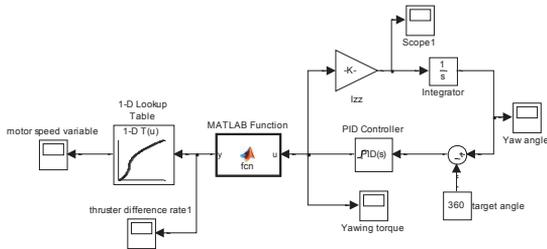


Figure 10: Simulink block for the rolling control system

In this part, the yaw angle rate target is set to the  $180^\circ$  of rolling angle. The PID controller was tuned and the value of Controller parameters for PID; proportional = -0.08, integral = -0.0007, derivative = 0.014 and filter coefficient = 2.1. Based on the result, the control system required less than 2s to reach  $180^\circ$  yawing angle target. The result is as shown in Figure 11.

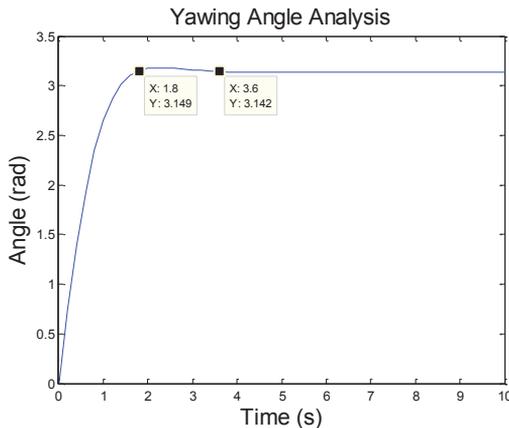


Figure 11: Yawing controller step response for  $180^\circ$  angle target.

Besides that, the motor speed variable also should be to concern in the analysis. The analysis used to observe the capability of the actuator to receive input signal regarding on the controller system requirement. The motor speed variable response as shown on Figure 12. Base on the response, maximum speed of change is the 169.5rad/s. If the initial condition of the system is maintain the altitude with 400rad/s speed of the motor, it mean the maximum motor speed in the system is 569.5rad/s, and it still under the breakage speed of the motor.

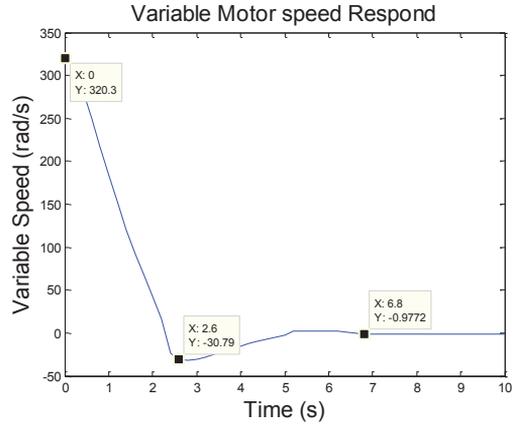


Figure 12: Rate of motor speed variable.

#### IV. QUADCOPTER ACCELERATION CONTROL

The control system is designed to produce the desired acceleration of the attitude control autonomously. Both angles of rolling,  $\theta$  and pitching,  $\phi$  would produce torque to generate movement based on the E-frame which each movement is about straightway fly and sideways fly. The acceleration controller of quadcopter movement is a major part in the development of an autonomous Quadcopter. The controller is used to ensure the movement is corresponded with the specified algorithm task. The control system constructed based on the simplification of mathematical model that has been derived. Because of the movement concept of X direction similar to the Y direction, so that only one movement will be described in this paper, which is the direction of movement. The quadcopter linear equation for x-axis will be analyzed as follows,

$$\ddot{X} = (\sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi) \frac{U_1}{m} \quad (9)$$

The equation can be simplified to make the system easy to analyze. Since the experiment only about the x-axis movement analysis, the change of angle value for yawing,  $\psi$  and pitching,  $\phi$  assumed as to be zero. Thus the quadcopter linear equation can be simplified as follows,

$$\ddot{X} = (\sin \theta) \frac{U_1}{m} \quad (10)$$

The angle of  $\theta$  will be obtained from rolling control system described in the section A of Attitude of quadcopter. Once again, PID controller will be involved to control the acceleration base on the target distance. By assuming the initial condition of the system is stationary flight, the controller simulation block is shown in Figure 13.

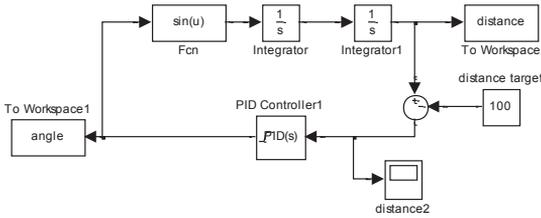


Figure 13: Simulation block design of acceleration control

Run the system with PID Controller parameters set up for; proportional constant = 1, integral constant = 1, derivative constant = 1 and filter coefficient = 100. The PID controller also needs to set their angle cut-off of  $30^\circ$  in the PID function block parameter. The step response of the simulation work is as shown in Figure 14. Based on the response, the system takes 25.95s to travel to a 100m distance.

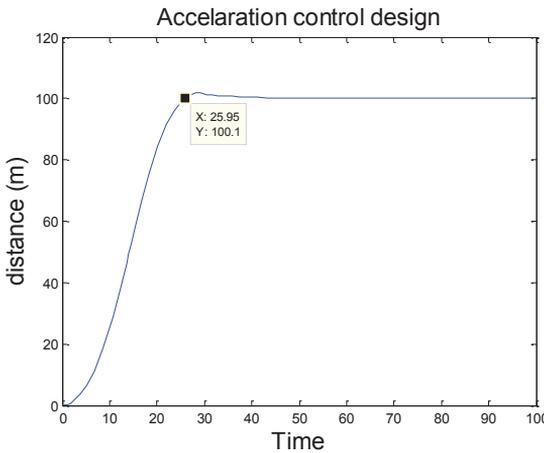


Figure 14: System response for the 100m distance

Since the system includes the angle of  $\theta$ , the quadcopter linear movement controller will be designed by involving the rolling control. The combination of both controllers has been conducted by the simulation in Matlab/Simulink and the controller designed as shown in Figure 15. Current rolling angles from the Rolling controller will be inserted into the acceleration controller as the current angle of the system, and the acceleration controller generates the target rolling angle will be used to the rolling controller as the reference angle target.

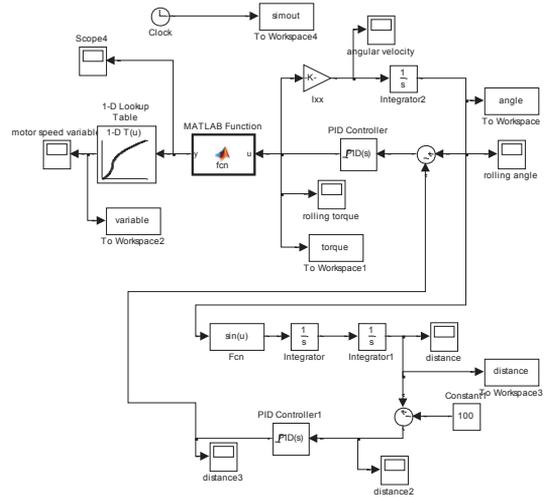


Figure 15: Combination of the rolling controller and the acceleration controller

The simulation is about the movement for 100m distance to the right sideway flight. The step response of the combination controller as shown in Figure 15 Based on the step response on Figure 16, the system take 26.53s to travel to a 100m distance.

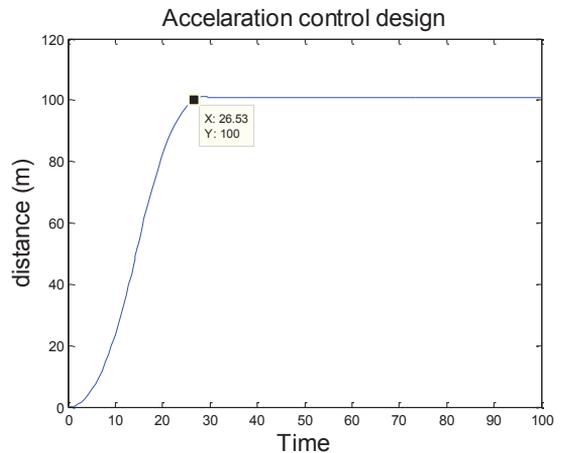


Figure 16: Step response for the combination controller

The controller system is also evaluated by comparing the current rolling angle and the targeting angle that is generated by the acceleration control system. The comparison for both response is in term of their time delay and the steady state error. Both responses are as shown in Figure 17 and Figure 18.

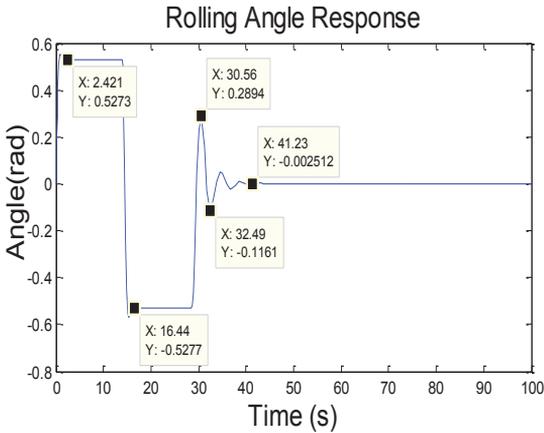


Figure 17: Current rolling angle of the system

Current rolling angle represents the measured angle of the real system, while the actual rolling angle target is the controller target angle that generated by the acceleration controller.

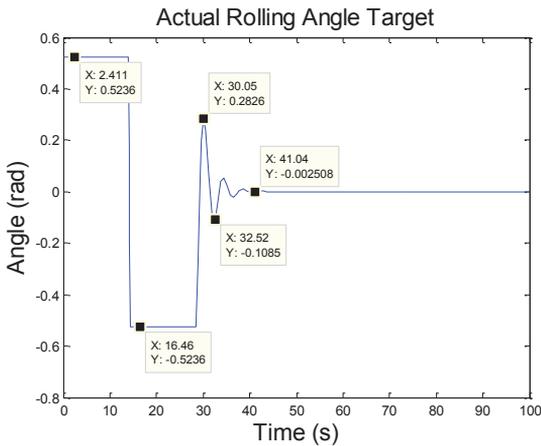


Figure 18: Actual rolling angle target

Based on both responses, the system shows a good synchronization on their feedback response to achieve the 100m distance target. Besides that, the motor speed variable also should be concerned in the analysis. The analysis used to observe the capability of the actuator to receive an input signal regarding on the controller system requirement. The motor speed variable response as shown on Figure 19. Based on the response, maximum speed of change is the 152.3rad/s. If the initial condition of the system maintains the altitude with the 400rad/s speed of the motor, it means the maximum motor speed in the system is 552.3rad/s, and it's still under the breakage speed of the motor.

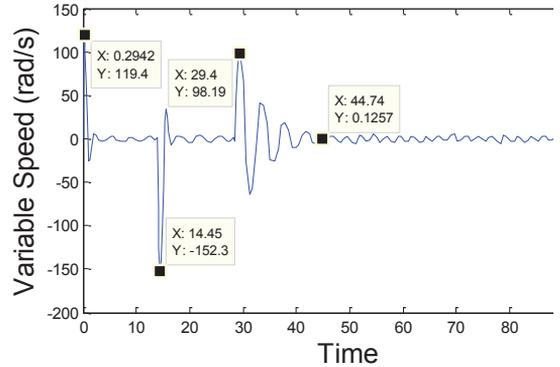


Figure 19: motor speed variable response

## V. CONCLUSION

The analysis of the attitude quadcopter system can be realized using relevant model description. The result of analysis helps others to start designing a controller that is compatible with the quadcopter system. Each model needs to be analyzed by undergoing simulation to understand the behavior of a system. The systematic procedure for designing a controller for quadcopter system has been presented here. In the future, all the subsystems of automatic controller will be combined together to form the complete autonomous quadcopter UAV control system. Thus, the system can be implemented into real application of an autonomous quadcopter UAV.

## ACKNOWLEDGMENT

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