

A Study on Forward and Inverse Problems for an Ultrasonic Tomography

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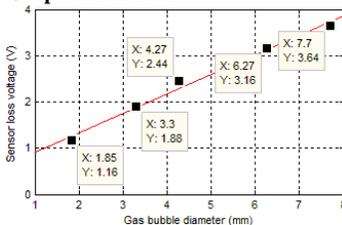
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Graphical abstract



Abstract

This paper studies the solution for inverse and forward problems for an ultrasonic tomography. Transmission-mode approach has been used for sensing the liquid/gas two-phase flow, which is a kind of strongly inhomogeneous medium. The imaging technique for two-phase flow using fan-shaped beam scanning geometry was presented. In this work, the tomographic images are derived from Back-Projection Algorithm. Some of the results based on the Linear Back-Projection algorithm (LBP) and the Hybrid Reconstruction algorithm (HR) was also presented.

Keywords: Ultrasonic; forward problem; inverse problem; image reconstruction

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1.0 INTRODUCTION

Process tomography can be used to obtain both qualitative and quantitative data needed in modelling a multi-fluid flow system [1]. The modelling is carried out to predict the spatial and temporal behaviour of a process and it becomes more significant as the inherent complexity of a process increases [2]. On the other hand, the transducer configuration is a key factor in the efficiency of data acquisition; it has both static and dynamic characteristics [3]. The static characteristics are the fundamental parameters which determine the physical structure of the configuration. Figure 1 show the configuration used in this study.

This system employs 16 pairs of ultrasonic transducer with 8.2 mm in diameter for each transducer. Both ultrasonic transmitters (Tx1-Tx16) and receivers (Rx1-Rx16) have a divergence angle, $\alpha = 125^\circ$ and were arranged around the circumference of the experimental pipe.

2.0 THE FORWARD PROBLEM

The forward problem determines the theoretical output of each of the sensors when the sensing area is considered to be two-dimensional [4]. The cross-section of the pipe is mapped onto a 64 by 64 rectangular array consisting of 4096 pixels as shown in Figure 2. The forward problem can be solved by using the analytical solution of sensitivity maps which produces the

sensitivity matrices [5]. Each transmitting sensors is virtually excited and the affected pixels are taken into account. Calculation of the sensitivity maps are outlined in the following section.

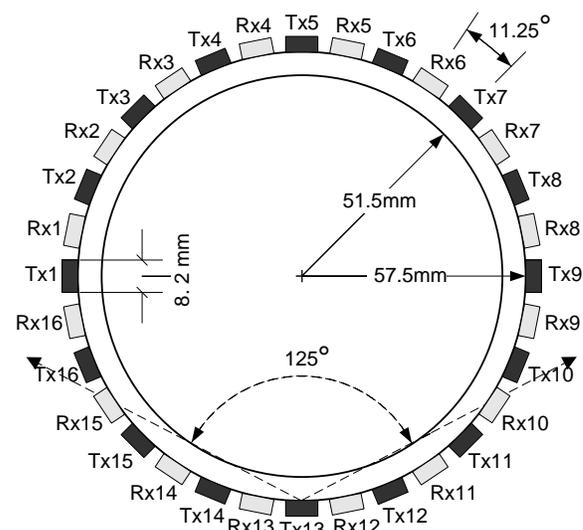


Figure 1 The measurement section configuration

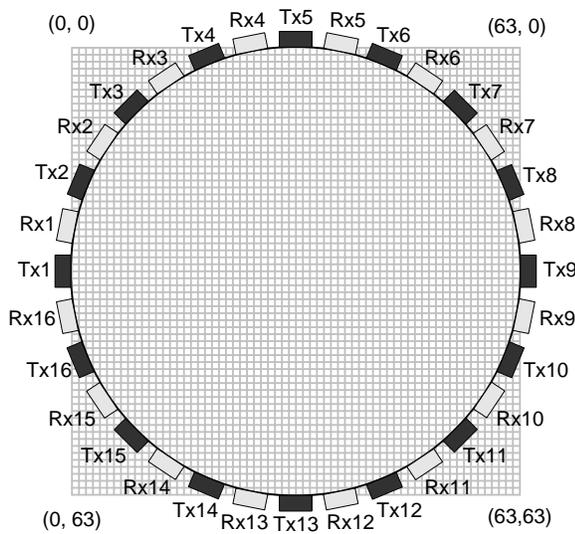


Figure 2 Image plane model for 64 x 64 pixels tomogram

3.0 THE SENSITIVITY MAPS

A plot of sensitivity distribution is called sensitivity map. The sensitivity distribution can be determined by calculating the ultrasonic energy attenuation at position of each receiver due to obstruction in the object space [6]. To create sensitivity maps, a model of measurement section has been developed. The measurement section model is divided into 256 nodes to create the round image plane model with 256 pixels in radius, r . Each node is separated by an angle, θ of 1.4063° . The transducer diameter or transducer arc (S_d) is represented by the seven red nodes on the image plane model. This is shown in Figure 3.

The image plane model in Figure 2 is developed by using 512 x 512 pixels. Then, this size is reduced to 64 x 64 pixels by grouping the 512 x 512 pixels into 8 x 8 pixels each. This is shown in Figure 4. To generate a series of sensitivity map, custom created software of Visual Basic 6.0 had been used. The projection of each transmitter to the receiver is represented by the virtual projection developed by using the Visual Basic 6.0 program. The illustration of virtual projection for projection of Tx13 to Rx7 is shown in Figure 4.

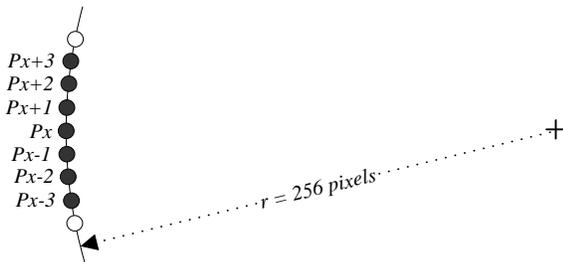


Figure 3 Nodes representing transducer arc on the image plane model

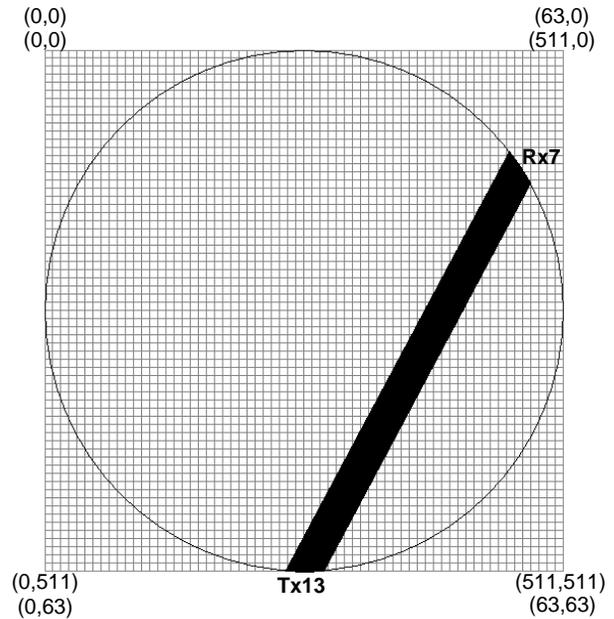


Figure 4 The virtual projection for Tx13 to Rx7

The virtual projection that lay on the projection path was coloured black. The computer graphic memory is used to retrieve the small pixels (512 x 512 pixels) colour occupied by the projection using the function provided by *Windows API* function call library. Any small pixels occupied by projection (blacked) is counted and summed into the corresponding major pixels (64 x 64 pixels). The result for projection in Figure 4 was then formed into a matrix. During the image reconstruction process, normalized sensitivity map has been used to ease the coordination of the colour level on the tomogram.

4.0 THE INVERSE PROBLEM

The inverse problem is to determine from the system response matrix (sensitivity matrices), a complex transformation matrix for converting the measured sensor values into pixel values. It is known as the tomogram [5]. The details for tomogram reconstruction are presented in the following section.

4.1 Image Reconstruction Algorithms

To reconstruct the cross-section of image plane from the projection data, back-projection algorithm has been employed. Most of the work in process tomography has focused on the back-projection technique. It is originally developed for the X-ray tomography and it also has the advantages of low computation cost [7]. The measurements obtained at each projected data are the attenuated sensor values due to object space in the image plane. These sensor values are then back projected by multiply with the corresponding normalized sensitivity maps. The back projected data values are smeared back across the unknown density function (image) and overlapped to each other to increase the projection data density. The process of back-projection is shown in Figure 5 and Figure 6.

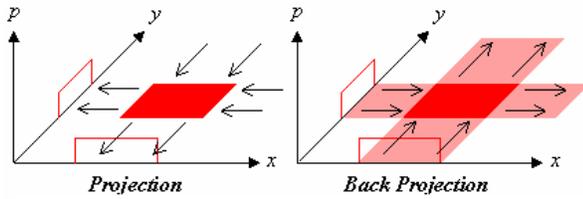


Figure 5 The back-projection method

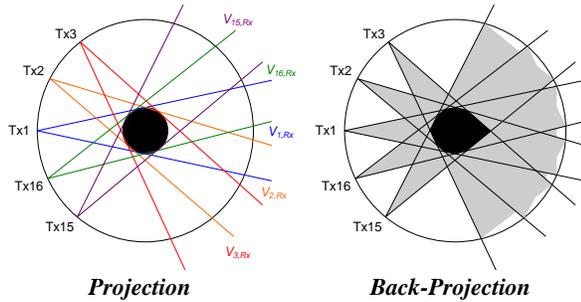


Figure 6 The fan-shaped beam back-projection

The density of each point in the reconstructed image is obtained by summing up the densities of all rays which pass through that point. This process may be described by Equation (1) [6]. Equation 1 is the back-projection algorithm where the spoke pattern represents blurring of the object in space.

$$f_b(x, y) = \sum_{j=1}^m g_j(x \cos \theta_j + y \sin \theta_j) \Delta \theta \quad (1)$$

where $f_b(x, y)$ is the function of reconstructed image from back-projection algorithm, θ_j is the j -th projection angle and $\Delta \theta$ is the angular distance between projection and the summation extends over all the m projection. Based on Equation (1), two image reconstruction algorithms were developed namely the Linear Back-Projection algorithm (LBP) and the Hybrid Reconstruction algorithm (HR).

4.2 Linear Back-projection Algorithm (LBP)

In Linear Back-projection algorithm (LBP), the concentration profile is generated by combining the projection data from each sensor with its computed sensitivity maps [8]. The modelled sensitivity matrices are used to represent the image plane for each view. To reconstruct the image, each sensitivity matrix is multiplied by its corresponding sensor loss value; this is same as back project each sensor loss value to the image plane individually [8]. Then, the same elements in these matrices are summed to provide the back projected voltage distributions (concentration profile) and finally these voltage distributions will be represented by the colour level (coloured pixels). This process can be expressed mathematically as below [9]:

$$V_{LBP}(x, y) = \sum_{Tx=1}^{16} \sum_{Rx=1}^{16} S_{Tx, Rx} \times \overline{M}_{Tx, Rx}(x, y) \quad (2)$$

where $V_{LBP}(x, y)$ is the voltage distribution obtained using LBP in the concentration profile matrix, $S_{Tx, Rx}$ is the sensor loss voltage for the corresponding transmission (Tx) and reception (Rx) and $\overline{M}_{Tx, Rx}(x, y)$ is the normalized sensitivity map for the view of Tx to Rx.

4.3 Hybrid Reconstruction Algorithm (HR)

The Hybrid Reconstruction algorithm (HR) is based on the previous development by Ibrahim [10]. This algorithm determines the condition of projection data and improves the reconstruction by marking the empty area during image reconstruction. As a result, the smearing effect caused by the back-projection technique is reduced. The projection data obtained by Ibrahim [10] is based on the sensor value. Later, Rahim et al. [6] had used a different approach where he used the signal loss measurement instead of direct projection data in order to reconstruct the fan-shaped beam image through optical technique. He claimed that this method is easier to implement compared to the original method.

The HR is obtained by multiplying the concentration profile obtained using the LBP with the HR masking matrix. The HR masking matrix was obtained by filtering each of the concentration profile element. If the concentration profile element is larger or equal to $\frac{3}{4}$ of the maximum pixel value, then the masking matrix element for the corresponding concentration profile element is set to one otherwise it is set to zero. The mathematical model for HR is shown as below:

$$V_{HR}(x, y) = B_{HR}(x, y) \times V_{LBP}(x, y) \quad (3)$$

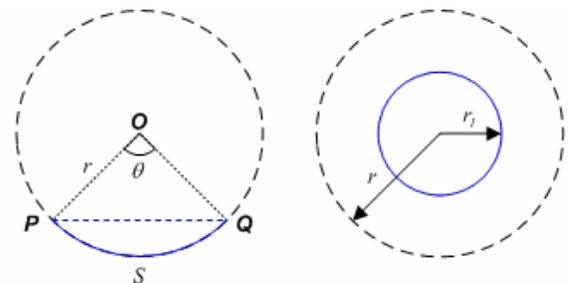
in which:

$$\begin{aligned} B_{HR}(x, y) &= 0 \Rightarrow V_{LBP}(x, y) < P_{Th} \\ B_{HR}(x, y) &= 1 \Rightarrow V_{LBP}(x, y) \geq P_{Th} \end{aligned} \quad (4)$$

where $B_{HR}(x, y)$ is the HR masking matrix, P_{Th} is the pixel threshold value ($\frac{3}{4}$ of the maximum value), $V_{LBP}(x, y)$ is the reconstructed concentration profile using LBP and $V_{HR}(x, y)$ is the improved concentration profile using HR algorithm.

5.0 RECONSTRUCTION ALGORITHM SIMULATION

In order to justify the quality of a reconstructed image, a standard phantom flow pattern is required so that it can be compared directly with the reconstructed image [11]. Several forward modelling have been developed in order to quantify the image reconstruction algorithm. The forward models developed are to simulate the typical flow regimes in the experimental pipe by using different image reconstruction algorithm. There are two types of forward models carried out in this research namely the stratified flow and the annular flow which is as illustrated in Figure 7.



a) Stratified Flow Modelling b) Annular Flow Modelling

Figure 7 Stratified flow and annular flow model

For stratified flow forward models, three regimes have been created representing one quarter flow, half flow and three quarter flow whereas for annular flow forward models, three regimes have been created for representing a 21.6 mm diameter annular flow, a 42.2 mm diameter annular flow and a 60.5mm diameter annular flow. The forward model image reconstruction is based on the theoretical image reconstruction representing the typical flow regimes. The stratified flow forward models are created by adjusting the size of *SPQ* minor segment according to the regime sizes. Meanwhile, for annular flow forward models, they are created by varying the radius of *r_l* according to the annular flow models. By using Visual Basic 6.0 software, the forward model regimes are placed on the image plane to estimate the results. The results obtained by the forward models are presented in section 6.

6.0 SIMULATION RESULTS

Based on the previous modelling of forward models, the image reconstruction algorithms namely the Linear Back-Projection Algorithm and the Hybrid Reconstruction Algorithm were tested, and the results are shown in Figures 8(a) and 8(b). These algorithms have been tested with six different flow models representing the forward models. They are one quarter flow, half flow, three quarter flow, 27 mm-diameter annular flow, 42.2 mm-diameter annular flow and 60.5 mm-diameter annular flow. The results are shown respectively in the following:

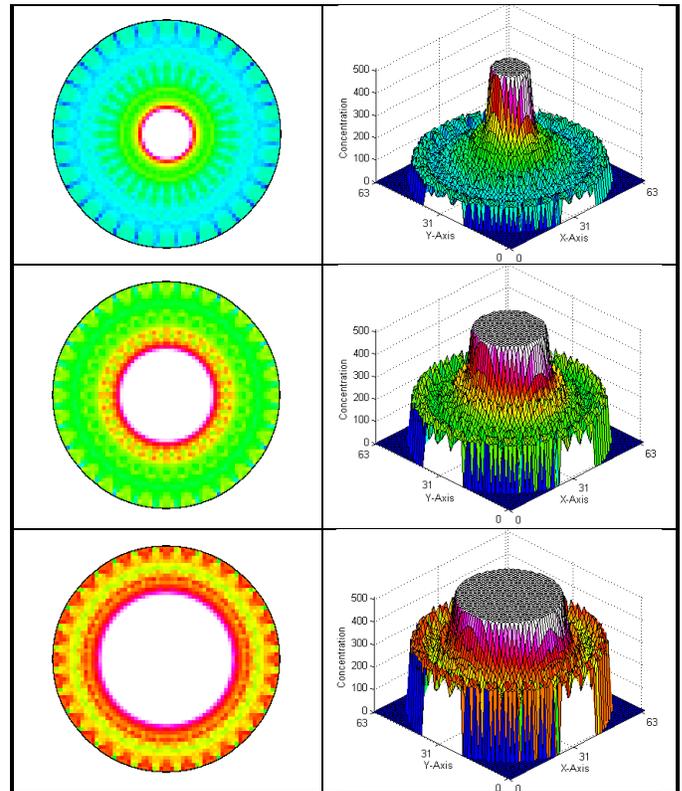
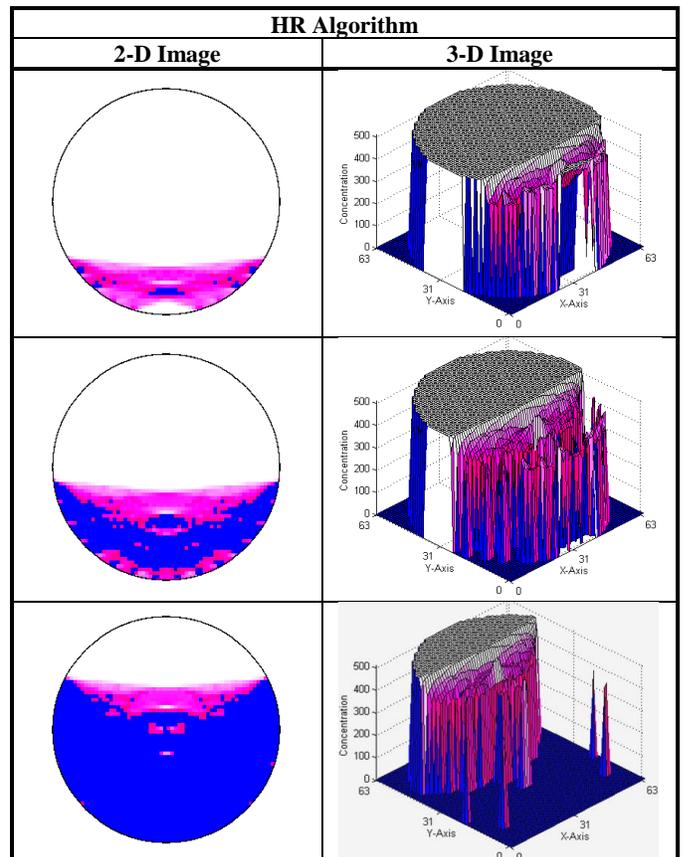
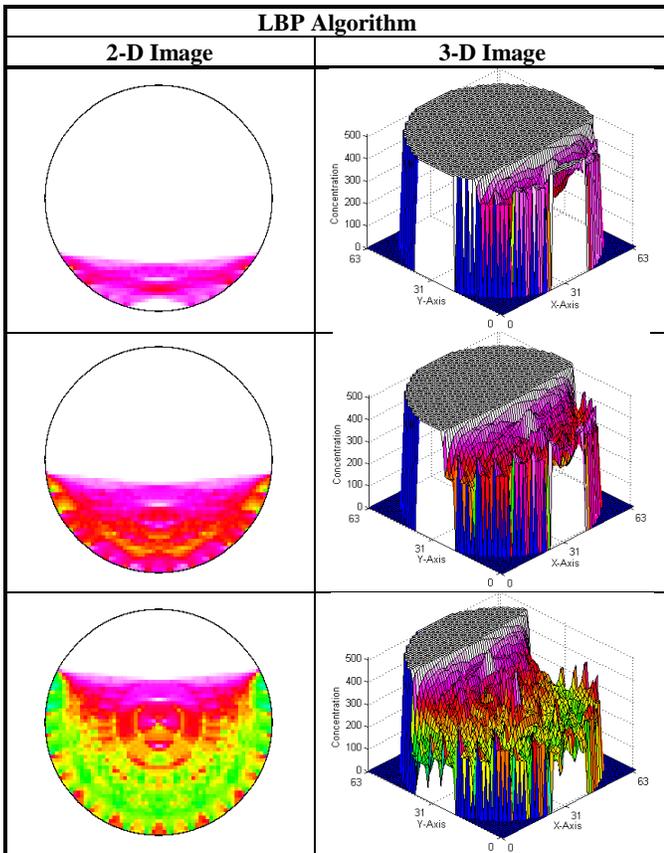


Figure 8(a) Simulation results using test profiles



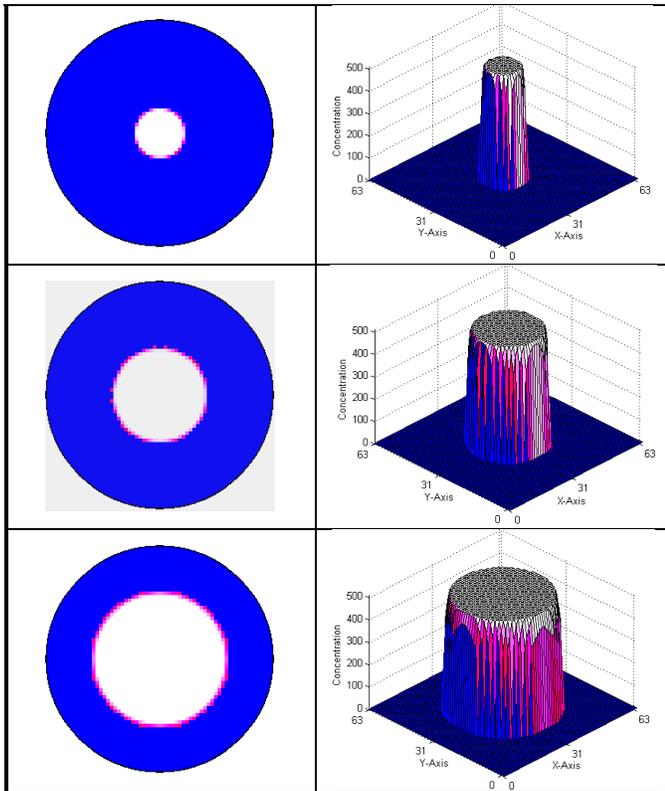


Figure 8(b) Simulation results using test profiles

7.0 DISCUSSIONS

Obviously, the back-projection technique results in blurring the object image. Reconstructed images by the forward models showed these blurring images except for the reconstruction by HR. As illustrated in Figure 5 and Figure 6, the blurring is due to the projection along straight lines. The intensity distribution is centre symmetrical and dependent on the projection angle where the blurring function is inversed of the corresponding pipe radius. Therefore, one of the methods to reduce the blurring is by using the HR method. From the forward model images, it show that the HR algorithm had successfully eliminates the spurious and blurring images and it corrects the reconstructed images by separating the object from the background.

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References

- [1] L. Xu, Y. Han, L.-A. Xu, and J. Yang. 1997. Application of Ultrasonic Tomography to Monitoring Gas/Liquid Flow. *Process Tomogr.* 52(13): 2171–2183.
- [2] R. Abdul Rahim, Y. Md. Yunos, M. H. Fazalul Rahiman, and H. Abdul Rahim. 2010. Mathematical Modelling of Gas Bubbles and Oil Droplets in Liquid Media Using Optical Linear Path Projection. *Spec. Issue Valid. Data Fusion Process Tomogr. Flow Meas. Valid. Data Fusion Process Tomogr. Flow Meas.* 21(3): 388–393.
- [3] N. M. Nor Ayob, M. J. Pusppanathan, R. Abdul Rahim, M. H. Fazalul Rahiman, and F. R. Mohd Yunus. 2013. Design Consideration for Front-End System in Ultrasonic Tomography. *J. Teknol.* 64(5): 53–58.
- [4] S. R. Arridge and J. C. Schotland. 2009. Optical Tomography: Forward and Inverse Problems. *Inverse Probl.* 25(12): 123010.
- [5] M. F. Rahmat, M. D. Isa, R. A. Rahim, and T. A. R. Hussin. 2009. Electrodynamics Sensor for the Image Reconstruction Process in an Electrical Charge Tomography System. *Sensors.* 9(12): 10291–10308.
- [6] R. Abdul Rahim, L. C. Leong, K. S. Chan, M. H. Rahiman, and J. F. Pang. 2008. Real Time Mass Flow Rate Measurement Using Multiple Fan Beam Optical Tomography. *ISA Trans.* 47(1): 3–14.
- [7] Mingquan Wang, Jinshuan Zhao, Shi Zhang, and Guohua Wang. 2010. Electrical Impedance Tomography Based on Filter Back Projection Improved by Means Method. Presented at the Biomedical Engineering and Informatics (BMEI), 2010 3rd International Conference on. 1: 218–221.
- [8] M. H. F. Rahiman, R. A. Rahim, and H. A. Rahim. 2012. Tomographic Reconstruction of a Multi-Attenuation Phantom by Means of Ultrasonic Method. In *Computer Science and Convergence.* 114. J. J. Park, H.-C. Chao, M. S. Obaidat, and J. Kim, Eds. Dordrecht: Springer Netherlands, 761–767.
- [9] M. H. F. Rahiman, R. A. Rahim, and M. Tajjudin. 2006. Ultrasonic Transmission-Mode Tomography Imaging for Liquid/Gas Two-Phase Flow. *Sens. J. IEEE.* 6(6): 1706–1715.
- [10] S. Ibrahim. 2000. *Measurement of Gas Bubbles in a Vertical Water Column Using Optical Tomography.* Sheffield Hallam University, UK. Ph.D. Thesis.
- [11] C. G. Xie, S. M. Huang, C. P. Lenn, A. L. Stott, and M. S. Beck. 1994. Experimental Evaluation of Capacitance Tomographic Flow Imaging Systems Using Physical Models. *Circuits Devices Syst. IEE Proc.* 141(5): 357–368.