International Journal of Mechanical and Materials Engineering (IJMME), Vol. 4 (2009), No. 3, 309-315

IDENTIFICATION OF ISOMORPHISM AMONG KINEMATIC CHAINS AND INVERSIONS USING LINK ADJACENCY VALUES

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ABSTRACT

The present work deals with problem of detection of isomorphism which is frequently encountered in structural synthesis of kimematic chains. Using the link adjacency values a new method has been proposed to reveal simultaneously chain is isomorphic and link is isomorphic. Two new invariants for each link, called as first adjacency link value [FALV] and second adjacency link value [SALV] are developed for identifying distinct mechanisms of a planar kinematic chain. Another two invariants maximum first adjacency link value and maximum second adjacency link value which are the by- product of the same method has been proposed to detect isomorphism among kinematic chains. These invariants takes into account the degree of links and type of joints of the kinematic chain and are used as the composite identification number of a kinematic chain and mechanisms. The proposed method is easy to compute, reliable and capable of detecting isomorphism in all types of kinematic chains. This study will help the designer to select the best kinematic chain and mechanisms to perform the specified task at conceptual stage of design. Some examples are provided to demonstrate the effectiveness of this method.

Keywords: Kinematic chain (KC), distinct mechanism (DM), joint value, link value.

1. INTRODUCTION

Detection of isomorphism between kinematic chains and among inversions of a given chain is one of the most important and challenging problem in structural synthesis of kinematic chain. Undetected isomorphism's results in duplicate solutions and an unnecessary effort where as falsely identified isomorphism eliminates possible candidate for new mechanisms. A lot of time and effort has been devoted to develop a reliable and computationally efficient technique. However, scope exists for an efficient and reliable method to detect isomorphism between kinematic chain and among inversions of a given chain. This paper is an attempt in this direction. Characteristic polynomial method (Uicker & Raicu, 1975), have the disadvantage of dealing with large numerical and later counter examples were also reported. Quist and Soni, (1971) presented a method which utilizes the loops of a chain for the identification of the chain. CPAM was incorporated as the index of isomorphism in the computerization of structural synthesis method based on transformation of binary chains (Mruthyunjaya, 1984a; 1984b; 1984c). Yan and Hawng (1984) defined the linkage path code of a kinematic chain and proposed it as a means to identify the chain. Agrawal and Rao (1985) proposed variable permanent function to identify kinematic chains. Code approaches (Ambekar & Agarwal, 1987) requires highly sophisticated algorithms and greater computational effort when applied to large kinematic chain. Rao and Raju (1991) proposed the hamming value of a link as an invariant index to detect mechanism. The canonical numbering scheme (Shin & Krishnamurthy, 1992) is capable of unique relabeling of the links of a given kinematic chain. However it tends to become computationally inefficient when there are higher number of symmetry group elements in the kinematic chain. Tuttle et al. (1989) proposed a method based on the theory of finite symmetry groups, for deriving distinct inversions of chain. Chu and Cao (1994) proposed that a link's CACT can be used as an index to distinguish inversions derived from a chain. Tang and Liu (1993) presented a method based on degree code as mechanism identifier. A method based on artificial neural network (Kong et al., 1999) theory was also investigated. Chang et al. (2002) presented a new method based on eigenvalues and eigenvectors of adjacent matrices of chains. The method possesses the advantages of using standard matrix theory and adopting automatic computation techniques. However, the authors seem ambiguous about the key points of the methods and include some fundamental errors in their theory. Hasan and Khan (2006) presented a method based on degrees of freedom of kinematic pairs. Srinath and Rao (2006) developed a technique using the concept of correlation not only to detect isomorphism but also to reveal the kinematic characteristics such as parallelism, type of freedom, symmetry of the chain. Sunkari and Schmidt (2006) first time, establishes, reliability of the existing spectral techniques for isomorphism detection. Ding and Huang (2007; 2009) addresses the problem of isomorphism identification by finding a unique representation of graphs. The unique representation of the graph makes isomorphism identification easy and sketching and establishment of graph database feasible. It remains efficient even when the links of kinematic chains increase into the thirties. However, almost all the methods reported so far are based on first adjacency of links. First adjacency of a link deals with the links that are directly joined to it. These methods not taken care of higher order adjacency of links, so these methods do not specify necessary and sufficient conditions. In the present work this aspect is given more importance and two structural invariants [FALV] and [SALV] are developed to identify distinct inversions of a given chain. Another two invariants maximum first adjacency link value and maximum second adjacency link value which are the by- product of the same method has been proposed to detect isomorphism among kinematic chains.

2. DEFINITIONS OF TERMINOLOGY

The following definitions are to be understood clearly before applying this method. Various definitions with their abbreviations are given below.

(i) Degree of link : A numerical value for the link ,based on its connectivity to other links Therefore quaternary link has degree equal to four and ternary link has degree equal to three.

(ii) Joint Value: For a particular joint it is define as the ratio of algebric sum of degree of all the connected links to the number of links connected at the joint (type of joint). It is denoted by J_v.

$$\mathbf{J_v} = \frac{\sum \text{Degree of all the connected links}}{\text{Number of links connected at the joint}} \quad (1)$$

For example the joint values of the joint a, b, d of the chain shown in Figure 1 are $J_a = 2.5$, $J_b = 2.5$ and $J_d = 3.0$ respectively.

(iii) Link Value: For a link it is defined as the sum of joint values of all the joints of that link. For example the Link Values of the link 1 and 2 of the chain shown in Figure 1 are [$L_1 = (j_a + j_b + j_g = 2.5+2.5+3.0 = 8.0)$] and [$L_2 = (j_b + j_c = 2.5+ 2.5 = 5.0)$] respectively.

3. NEW STRUCTURAL INVARIANTS

Considering all essential features of the kinematic chains two new structural invariants [FALV] and [SALV] are proposed. The**y** are invariants for a kinematic chain mechanism because they are independent of the labelling of links and joints of a chain.

(i) First Adjacency Link Value [FALV]: For a link it is defined as the sum of link values of all the links that are directly joined to it. For example the First Adjacency Link Values of the link 1 and 2 of the chain shown in figure 1 are [$L_{1f} = (L_2 + L_6 + L_7 = 5.0 + 4.5+ 8.5 = 18.0)$] and [$L_{2f} = (L_1 + L_3 = 8.0+ 8.0 = 16)$] respectively.

(ii) Second Adjacency Link Value [SALV]: For a link it is defined as the sum of First Adjacency link values of all the links that are directly joined to it. For example the Second Adjacency Link Values of the link 1 and 8 of the chain shown in figure 1 are [$L_{1s} = (L_{2f} + L_{6f} + L_{7f} = 16 + 12.5+21 = 49.5)$] and [$L_{8s} = L_{7f} + L_{9f} = 21.0 + 12.5 = 33.5)$] respectively.

(iii) Maximum First Adjacency Link Value [FALVmax]: For a kinematic chain it is defined as the maximum of first adjacency link value of all the links of candidate kinematic chain.

(iv) Maximum Second Adjacency Link Value [SALVmax]: For a kinematic chain it is defined as the maximum of second adjacency link value of all the links of candidate kinematic chain.

4. PROPOSED TEST – BASIS

The kinematic chains are complex chains of combinations of binary ternary, and the other higher order links. These links are joined together by simple or multiple joints. While considering structural equivalence it is essential to consider type of links, joints and layout of the links in the assembly. According to graph theory, one essential condition for isomorphism is that the number and degree of vertices must be identical. In the pretext of chains it means that the number and type of links must be same. The type of links can be identified by their degrees, as it acts as their identification number. Thus a binary link has a value two, ternary three, quaternary four and so on. Degrees of links are used to assign values to joints and these are utilized to determine first and second adjacency values for all the links of a chain.

4.1 Test for isomorphism among inversions

The first and second link adjacency values of the various links have the potential to disclose how many inversions can be obtained from a given chain and by fixing which links these inversions are possible. If first and second link adjacency values of two links are identical then the inversions are equivalent and constitute only one distinct mechanism.

4.2 Test for isomorphism among kinematic chains

The proposed invariants [FALVmax] and [SALVmax] are a definitive test for isomorphism among chains. The test is simply based on comparing these two structural invariants. If these invariants are same then chains are said to be isomorphic, otherwise non isomorphic.

5. APPLICATION TO KINEMATIC CHAINS

Example-1 This example concerns with 9-links, 11-joints, and two degree of freedom kinematic chain shown in Figure 1.

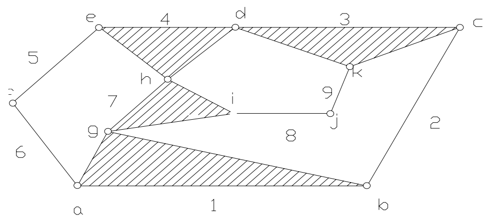

Figure1 Nine link two degree of freedom kinematic chain.

For chain 1:

Joint Values: The joint values of the different joints are as follows: $J_a = (3+2) / 2 = 2.5$, $J_b = (3+2) / 2 = 2.5$, $J_c = (3+2) / 2 = 2.5$, $J_d = (3+3) / 2 = 3.0$, $J_e = (3+2) / 2 = 2.5$, $J_f = (2+2) / 2 = 2.0$, $J_g = (3+3) / 2 = 3.0$, $J_h = (3+3) / 2 = 3.0$, $J_i = (2+3) / 2 = 2.5$, $J_j = (2+2) / 2 = 2.0$, $J_k = (3+2) / 2 = 2.5$

Structural invariants of the mechanisms: The structural invariants of the mechanisms derived from the above kinematic chain are listed below in Table 1.

Identification of the distinct mechanisms: Observing the structural invariants for the above nine mechanisms, it is found that the links- (1, 3), (4, 7), (5, 8) and (6, 9) are equivalent links as their structural invariants are same and forms four Distinct mechanism. Link 2 has the distinct invariants, forms the fifth distinct mechanism. Therefore, 5 distinct mechanisms are obtained from chain shown in figure 1.

Example-2 The second example concerns two KC with 12 links, 16 joints, single-degree of freedom co-spectral graph as shown in Figure 2. The graphs having same characteristic polynomials derived from the (0, 1) adjacency matrix are called as co-spectral graphs. The task is to examine whether these two KC are isomorphic.

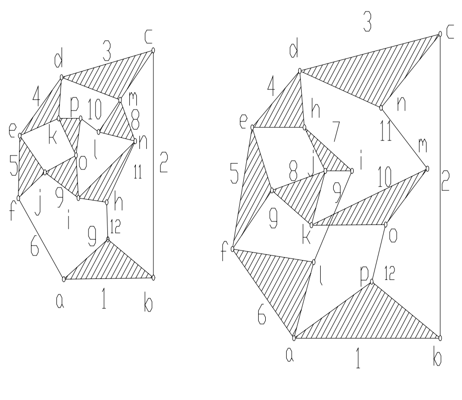

Figure 2 Twelve-Link single degree of freedom kinematic chains

For chain 2:

Joint Values: [$J_a = 2.5$, $J_b = 2.5$, $J_c = 2.5$, $J_d = 3$, $J_e = 2.5$ $J_f = 2.5$, $J_g = 3$, $J_h = 2.5$, $J_i = 3$, $J_j = 3$, $J_k = 3$, $J_l = 2.5$, $J_m = 3$, $J_n = 3$, $J_o = 3$, $J_p = 2.5$]

Link Values: [$L_1 = 7.5$, $L_2 = 5.0$, $L_3 = 8.5$, $L_4 = 9.0$, $L_5 = 8.5$, $L_6 = 5.0$, $L_7 = 8.5$, $L_8 = 8.5$, $L_9 = 9.0$, $L_{10} = 5.0$, $L_{11} = 8.5$, $L_{12} = 5.0$]

First adjacency link values: [15.0, 16.0, 22.5, 25.5, 23.0, 16.0, 23.0, 22.0, 25.5, 17.0, 22.5, and 16.0]
Second adjacency link values [48.0, 37.5, 63.5, 68.5, 67.0, 38.0, 68.0, 62.0, 68.5, 45.0, 63.5 and 37.5]

For chain 3:
Joint Values: [$J_a = 3$, $J_b = 2.5$, $J_c = 2.5$, $J_d = 3$, $J_e = 3$, $J_f = 3$, $J_g = 3$, $J_h = 3$, $J_i = 2.5$, $J_j = 3$, $J_k = 3$, $J_l = 2.5$, $J_m = 2.5$, $J_n = 2.5$, $J_o = 2.5$, $J_p = 2.5$].
Link Values: [$L_1 = 8.0$, $L_2 = 5.0$, $L_3 = 8.0$, $L_4 = 9.0$, $L_5 = 9.0$, $L_6 = 8.5$, $L_7 = 8.5$, $L_8 = 9.0$, $L_9 = 5.0$, $L_{10} = 8.0$, $L_{11} = 5.0$, $L_{12} = 5.0$].
First adjacency link values [18.5, 16.0, 19.0, 25.5, 26.5, 22.0, 23.0, 25.5, 17.0, 19.0, 16.0, and 16.0].
Second adjacency link values [54.0, 37.5, 57.5, 68.5, 73.0, 62.0, 68.0, 68.5, 45.0, 57.5, 38.0, and 37.5].
Example 3: The third example concerns another example of two kinematic chains with 10 links, 13 joints, single freedom as shown in Figure 3. The task is to examine whether these two chains are isomorphic.

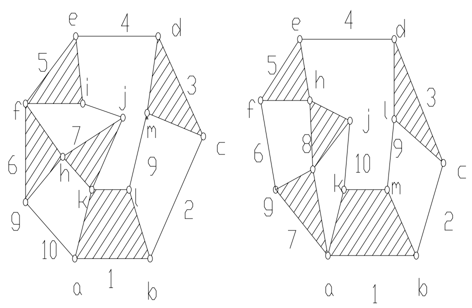

Figure 3 Ten Link Three degree of freedom kinematic chains

For chain 4:
Joint Values: [$J_a = 3$, $J_b = 3$, $J_c = 2.5$, $J_d = 2.5$, $J_e = 3$, $J_f = 3$, $J_g = 2.5$, $J_h = 3$, $J_i = 2.5$, $J_j = 2.5$, $J_k = 2.5$, $J_l = 3$].
Link Values: [$L_1 = 12.5$, $L_2 = 5.5$, $L_3 = 7.5$, $L_4 = 5.0$, $L_5 = 8.0$, $L_6 = 8.5$, $L_7 = 9.0$, $L_8 = 5.0$, $L_9 = 5.5$, $L_{10} = 5.5$].
First adjacency link values: [25.5, 20.0, 16.0, 15.5, 18.5, 22.5, 26.0, 17.0, 20.0, and 21.0].
Second adjacency link values: [87.0, 41.5, 55.5, 34.5, 55.0, 65.5, 65.0, 44.5, 41.5, and 48.0].

For chain 5:
Joint Values: [$J_a = 2.5$, $J_b = 2.5$, $J_c = 2$, $J_d = 2.5$, $J_e = 2.5$, $J_f = 2.5$, $J_g = 2.5$, $J_h = 2$, $J_i = 3$, $J_j = 3$, $J_k = 2.5$, $J_l = 2.5$]
Link Values: [$L_1 = 12.5$, $L_2 = 5.5$, $L_3 = 7.5$, $L_4 = 5$, $L_5 = 5$, $L_6 = 5.0$, $L_7 = 9.0$, $L_8 = 8.5$, $L_9 = 5.5$, $L_{10} = 5.5$]
First adjacency link values [25.5, 20.0, 16.0, 15.5, 18.5, 17.0, 26.0, 22.5, 21.0, and 20.0].
Second adjacency link values [87.0, 41.5, 55.5, 34.5, 55.0, 44.5, 65.0, 65.5, 48.0, and 41.5].

Structural invariants: The values of structural invariants for various chains are as follows:

For chain 2:
FALVmax = 25.5 and SALVmax = 68.5
For chain 3:
FALVmax = 26.5 and SALVmax = 73.0
For chain 4:
FALVmax = 26.0 and SALVmax = 87.0
For chain 5:
FALVmax = 26.0 and SALVmax = 87.0

This method reports that chain 2 and 3 are non-isomorphic as the corresponding values of FALVmax and SALVmax are different and chain 4 and 5 are isomorphic as the corresponding values of FALVmax and SALVmax are same. Note that by using another methods eigenvector (Chang et al., 2002) and artificial neural network (Kong et al., 1999) the same conclusion is obtained, proving reliability of this method. *Identification of the distinct mechanisms:* Observing the structural invariants of the various links, it is found that the both chain 2 and 3 have six identical links so 9 distinct mechanisms can be obtained from these chains. Similarly chains 4 and 5 both have two identical links so 9 distinct mechanisms can also be obtained from these chains.

Table 1 Structural invariants of various links

Link	Link Value	Adjacent Links	FALV	SALV
1	8.0	2,6,7	18.0	49.5
2	5.0	1,3	16.0	36.5
3	8.5	2,4,9	18.0	49.5
4	8.5	3,5,7	21.0	52.0
5	4.5	4,6	13.0	33.5
6	4.5	1,5	12.5	31.0
7	8.5	1,4,9	21.0	52.0
8	4.5	7,9	13.0	33.5
9	4.5	3,8	12.5	31.0

Table 2 Total Number of DM derived from 1F KC of 8 and 10 links

Class	Group	n	KC	DM	TDM
	a	8	9	35	
III	b	8	5	31	
	c	8	2	05	71
IV	a	10	50	327	
	b	10	95	859	
	c	10	15	123	
	d	10	57	406	
	e	10	8	68	
	f	10	3	20	1808
	h	10	2	05	

6. RESULTS

(i) The proposed composite structural invariants [FALVmax] and [SALVmax] of the KC are able to detect isomorphism among the KCs and even KCs with co-spectral graphs. All the simple jointed 1-F, 8-links 16 KC and 1-F, 10-links 230 KC along with 2-F, 9-links 40 KC have been tested successfully for their non-isomorphism.
(ii) Using the proposed method the number of mechanisms derived from the family of 1-F; 6-link, 8-link and 2-F 9 link chains are 5, 71 and 254 respectively. These results are in agreement those reported already in the literature. The distinct mechanisms derived from the family of 1-F; 10-link 1808. A brief summary of these results are listed in Table-2. As an illustration, a list of 254 inversions in case of 40 nine-link two degree of freedom chains is given in Appendix A along with the structural invariants of various links.

7. CONCLUSION

Though no proof has been offered in the present work, the authors strongly believe that this method is unique as it takes care of nature and all inherent properties of the mechanisms. The proposed method is applicable to planar chains of any size and complexity even the KC with co-spectral graphs and also able to identifying all distinct mechanisms derived from a given kinematic chain. It is hoped that the proposed method presents a new concept on which a new classification system for distinct mechanisms can be based. Such a new identification would be extremely selective and would minimize, if not completely eliminate the possibility of duplicate identification for structurally different mechanisms. The inherent relation between structural invariants and the mechanisms need further study.

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Appendix A [For chain no. ref. (Rao & Raju, 2000)]

Chain no.	Structural invariants of Various links (FALV,SALV)	Inversions	*TDM
1	(21.5,56),(21.5,56),(21.5,56.5),(21.5,56.5),(13,34.5),(13,34.5),(13.5,30.5),(13.5,30.5),(9,27)	(1,2), (3,4), (5,6),(7,8), 9	5
2	(21.5,55.5),(18.5,53.5),(21,52.5),(16.5,40),(18,49.5),(16,36.5),(12.5,30),(9,24.5),(12,27)	1,2,3,4,5,6,7,8,9	9
3	(18.5,53),(18.5,53),(16,36),(16,36),(21,52),(21,52),(12.5,30),(12.5,30),(9,25)	(1,2), (3,4), (5,6),(7,8), 9	5
4	(21.5,55.5),(21.5,55.5),(13,34.5),(13,34.5),(21.5,56),(21.5,56),(12.5,30.5),(9,25),(12.5,30.5)	(1,2), (3,4), (5,6),(7,9), 8	5
5	(18,50),(18,50),(16.5,39.5),(16.5,39.5),(21.5,56),(21.5,56),(12,27), (12,27), (9,24)	(1,2), (3,4), (5,6),(7,8), 9	5

6	(18.5,52),(18.5,52),(16,37.5),(16,37.5),(12,27.5), (12,27.5),(19,56),(24,56),(9,55)	(1,2), (3,4), (5,6),7,8, 9	6
7	(18.5,52.5),(21,52.5),(18,49.5),(16,36.5), (15.5,30.5),(16,35.5),(14.5,43.5),(12,27),(12.5,30)	1,2,3,4,5,6,7,8,9	9
8	(21,52),(21,52),(18,49.5),(18,49.5),(12.5,31), (12.5,31),(13,33.5),(13,33.5),(16,36)	(1,2), (3,4), (5,6),(7,8), 9	5
9	(21.5,53.5),(20.5,52.5),(18,49),(16,39.5), (17,46.5),(13,33),(12.5,30),(12.5,29.5),(12.5,30.5)	1,2,3,4,5,6,7,8,9	9
10	(18.5,52.5),(18.5,52.5),(12.5,31),(12.5,31), (12.5,30.5),(12.5,30.5),(18,49),(24,35),(16,37)	(1,2), (3,4), (5,6),7,8, 9	6
11	(21,52),(21,52),(13,34),(13,34),(18,49.5), (18,49.5),(12.5,30.5),(12.5,30.5),(16,36)	(1,2), (3,4), (5,6),(7,8), 9	5
12	(18.5,52),(18.5,52),(15.5,33),(15.5,33), (16,37),(20.5,50),(14.5,43),(12,27.5),(13,32.5)	(1,2), (3,4), 5,6,7,8, 9	7
13	(15.5,33.5),(15.5,33.5),(15,46.5),(18.5,43.5), (20.5,49.5),(15.5,33),(18,48.5),(12.5,31),(13,33)	(1,2), 3,4, 5,6,7,8, 9	8
14	(18,49.5),(18,49.5),(16,36),(16,36),(17.5,46.5), (17.5,46.5),(12.5,30),(12.5,30),(16,35)	(1,2), (3,4), (5,6),(7,8), 9	5
15	(17.5,46.5),(17.5,46.5),(16,35.5),(16,35.5), (18,49.5),(18,49.5),(12.5,30),(12.5,30),(16,36)	(1,2), (3,4), (5,6),(7,8), 9	5
16	(18,49),(18,49),(15.5,33),(15.5,33), (12.5,30.5),(12.5,30.5)	(1,2), (3,4), (5,6),7,8, 9	6
17	(18,49.5),(18,49.5),(12.5,31),(12.5,31),(13,33.5), (13,33.5),(21,52),(21,52),(16,36)	(1,2), (3,4), (5,6),(7,8), 9	5
18	(15.5,33),(15.5,33),(15.5,33),(15.5,33),(18,49), (18,49),(15,46),(15,46),(15,30)	(1,2,3,4), (5,6),(7,8), 9	4
19	(15.5,33),(15.5,33),(15.5,33),(15.5,33),(18,39), (18,39),(15,36),(15,36),(15,30)	(1,2,3,4), (5,6),(7,8), 9	4
20	(24.5,79),(19,58.5),(25,56.5),(20.5,43.5),(17,37.5),(13,36), (16.5,37),(9.5,29.5),(13,34.5)	1,2,3,4,5,6,7,8,9	9
21	(20,40.5),(20,40.5),(16,56),(24.5,78.5),(22,53),(16,38), (16.5,34),(9.5,29),(12.5,31.5)	(1,2), 3,4, 5,6,7,8, 9	8
22	(20,40.5),(20,40.5),(19,58),(21.5,76),(20,39.5), (18,51),(16,31),(9.5,28),(12,27.5)	(1,2), 3,4, 5,6,7,8, 9	8
23	(22.5,57),(22.5,57),(13,35.5),(13,35.5),(17,37),(17,37), (27,79),(17,45),(10,34)	(1,2), (3,4), (5,6),7,8, 9	6
24	(16,37.5),(16,37.5),(16.5,34),(16.5,34),(22.5,56),(24,68), (12.5,32),(15.5,44.5),(10,33)	(1,2), (3,4), 5,6,7,8, 9	7
25	(16.5,34.5),(16.5,34.5),(20.5,53.5),(12.5,32.5), (13.5,37.5),(25,57),(19,58),(24.5,78.5),(10,33)	(1,2), 3,4, 5,6,7,8, 9	8
26	(22.5,56.5),(17,45),(13.5,40),(17.5,40.5),(27,79.5), (22.5,57.5),(12.5,32),(9.5,29.5),(17,36.5)	1,2,3,4,5,6,7,8,9	9
27	(19,57.5),(20.5,43.5),(17,38.5),(14,42),(25,57.5), (24.5,79),(12,28.5),(9.5,28.5),(16.5,34)	1,2,3,4,5,6,7,8,9	9
28	(19.5,37.5),(19.5,37.5),(19.5,37.5),(19.5,37.5) (15.5,51),(15.5,51),(12,27.5),(12,27.5),(22,78)	(1,2,3,4), (5,6),(7,8), 9	4
29	(24,76),(22,53.5),(13.5,39),(17,37.5),(20,39), (16,37),(15,48.5),(12.5,32), (17,36.5)	1,2,3,4,5,6,7,8,9	9
30	(22,53.5),(22,53.5),(13.5,39.5),(13.5,39.5), (17.5,40.5),(17.5,40.5),(13,35),(13,35),(27,79)	(1,2), (3,4), (5,6),(7,8), 9	5
31	(17,37),(17,37),(13,35),(13,35),(17,38),(14,42), (18,51),(25,56),(24,75)	(1,2), (3,4), 5,6,7,8, 9	7
32	(19.5,37.5),(19.5,37.5),(16,54),(21.5,75), (19.5,36.5), (15,31),(15,47),(12.5,31.5),(16.5,34)	(1,2), 3,4, 5,6,7,8, 9	8
33	(20,39.5),(20,39.5),(15.5,52),(24,78.5),(21.5,49.5),(12,28), (12.5,33.5),(13,38.5),(17,37)	(1,2), 3,4, 5,6,7,8, 9	8
34	(20,40),(20,40),(16.5,34),(16.5,34),(13,33.5), (13,33.5), (17,45),(21,73),(19,57)	(1,2), (3,4), (5,6),7,8, 9	6
35	(18,51),(18,51),(20,39),(20,39),(16.5,34),(16.5,34) (13,34.5),(13,34.5),(21,73)	(1,2), (3,4), (5,6),(7,8), 9	5
36	(22,81),(22,81),(24,44),(24,44),(17,39),(17,39),(16,32), (16,32),(10,32)	(1,2), (3,4), (5,6), 7,8), 9	5

37	(17,38),(17,38),(17,38),(17,38),(17,38),(17,38), (21,75), (21,75),(24,42)	(1,2,3,4,5,6),(7,8), 9	3
38	(18,46),(18,46),(18,46),(18,46),(28,81),(28,81), (17,38), (17,38), (10,34)	(1,2,3,4), (5,6),(7,8), 9	4
39	(25,45),(25,45),(21.5,39.5),(21.5,39.5),(28.5,115) (22,41.5),(13,38.5),(16.5,63),(11,43)	(1,2), (3,4), 5,6,7,8, 9	7
40	(22.5,45.5),(22.5,45.5),(14.5,49.5),(14.5,49.5) (22,42),(22,42),(27,60),(31,116),(11,44)	(1,2), (3,4), (5,6),7,8, 9	6
	*TDM = Total distinct mechanism	*Grand Total	254