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MODELING OF FATIGUE CRACK GROWTH UNDER CONSTANT AND VARIABLE AMPLITUDE LOADING

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ABSTRACT

In this paper, a modeling of fatigue crack growth of thin walled tube aluminum alloy with circumferential crack under constant and variable amplitude loading are presented. Three Fatigue crack growth models, namely Walker, Forman and NASGRO were examined. The results showed that the simulation of the fatigue crack growth under constant and variable amplitude loading in the proposed technique is an efficient tool and also the different models gave different fatigue crack growth behavior. There are many factors affecting the fatigue crack growth in structures. Crack initiation and stress ratio are studied and shown a great influence on the life of the thin walled tube. The NASGRO model was found to be the most appropriate model for the variable amplitude loading. Therefore, this model may be suggested for use in critical applications in studying fatigue crack growth for different structures under variable amplitude loading.

Keywords: Crack initiation, Constant and variable amplitude loading, Fatigue crack growth, Modeling, stress ratio and thin wall tube.

INTRODUCTION

Crack growth in structures depends on the amplitude, stress ratio, and frequency of the load. Due to the random nature of variable loading, it is difficult to model all these influential parameters correctly. Overloads are known to retard crack growth, while underloads accelerate crack growth relative to the background rate. These interactions, which are highly dependent upon the loading sequence, make the prediction of fatigue life under variable amplitude loading more complex than under constant amplitude loading. Many models have been developed to predict the fatigue life of components subjected to variable amplitude loading (Paris *et al.*, 1999; Sadananda and Vasudevan, 1999; James and Paterson, 1997; Taheri, 2003). The earliest of these are based on calculations of the yield zone size ahead of the crack tip and are still widely used. The Wheeler model (Wheeler, 1972) and Willenborg *et al.*, (1997) model, for example, both fall into this category. Another category models based on the crack closure approach, which considers plastic deformation and crack face interaction in the wake of the crack, was subsequently proposed by Elber (1972) have been used to model crack growth rates under variable amplitude loads (Newman, 1984; Ray and Patanker, 2001; Newman *et al.*, 2001; Schijve, 1981). More recent proposals include combinations of the Wheeler model with the Newman crack closure model (Voorwald and Torres, 1991) and model based on the strain energy density factor (Huang *et al.,* 2005). However, due to the number and complexity of the mechanisms involved in this problem, no universal model exists yet. Kujawski (2005) clearly indicated that there is no general agreement among researchers regarding the significance of closure concept on fatigue crack behavior.

In the present work, a fatigue crack growth (FCG) of thin walled aluminum alloy with circumferential crack under constant and variable amplitude loading (CAL and VAL) in tension was studied to show the behavior of FCG with different load history. The FCG models Walker, Forman and NASGRO gives different behavior under the two types of loading. There are many factors affecting the FCG. The effects of initial crack length and stress ratio on crack growth are shown with great influence. Need to study the other factors such load interaction, environments, crack angle etc.

THEORETICAL BACKGROUND

Although a general understanding of many aspects of fatigue crack growth behavior was established in the early 1960s, a specific accumulation of damage model for computation of growth under a wide variety of service loads was lacking. The reason for building models is to link theoretical ideas with the observed data to provide a good prediction of future observations. Modeling of FCGR data has enhanced the ability to create damage tolerant design philosophies.

The first paper was published by Paris *et al,* (1961) and it turned out to be a milestone publication. The major limitation of the Paris law is its inability to account for the stress ratio. This drawback notified Walker (1970) to improve the Paris model by including the effect of stress ratio. Walker proposed a parameter ΔK, which is an equivalent zero to maximum (R = 0) stress intensity factor that causes the same growth rate as the actual K_{max}, and R combination. It is expressed by the relationship of

$$\overline{\Delta K} = K_{\max}\left(1-R\right)^{\gamma_W} \qquad (1)$$

Where K_{max}= Δ K/(1-R), and Equation (1) becomes

$$\overline{\Delta K} = \frac{\Delta K}{\left(1-R\right)^{1-\gamma_W}} \qquad (2)$$

Therefore, the Walker law is represented by

$$\frac{da}{dN} = C_W\left(\overline{\Delta K}\right)^{m_W}$$

or

$$\frac{da}{dN} = C_W\left[\frac{\Delta K}{\left(1-R\right)^{1-\gamma_W}}\right]^{m_W} \qquad (3)$$

For R = 0, Equation (3) is formulated to form

$$\frac{da}{dN} = C_W\left(\Delta K\right)^{m_W} \qquad (4)$$

Which, is equivalent to the Paris law with

$Cp = C_w$ and $m_p = m_w$.

Although Walker improved the Paris model by taking account of the stress ratio, neither models could account for the instability of the crack growth when the stress intensity factor approaches its critical value. Forman (1972) improved the Walker model by suggesting a new model, which is capable of describing region III of the fatigue rate curve and includes the stress ratio effect. The Forman law is given by the mathematical relationship

$$\frac{da}{dN} = \frac{C_F\left(\Delta K\right)^{m_y}}{\left(1-R\right)K_C - \Delta K} = \frac{C_F\left(\Delta K\right)^{m_y}}{\left(1-R\right)\left(K_C - K_{\max}\right)} \qquad (5)$$

where K_c is the fracture toughness for the material and thickness of interest. Equation (5) indicates that as K_{max}, approaches K_c & da/dN tends to infinity. Therefore, the Forman equation is capable of representing stable intermediate growth (region II) and the accelerated growth rates (region III) shown in Figure 1. The Forman equation is capable of representing data for various stress ratios by computing the following quantity for each data point, i.e.

$$Q = \frac{da}{dN}\left[\left(1-R\right)K_C - \Delta K\right] \qquad (6)$$

If the various Δ K and R combinations fall together on a straight line on a log-log plot of Q versus Δ K, the Forman equation is applicable and may be used. Comparing Equations (5) and (6), the Forman equation can be represented as

$$Q = C_F\left(\Delta K\right)^{m_y} \qquad (7)$$

The Forman equation is capable of representing data of various stress ratios for region II and III. Further modifications of the Forman's expression to represent region I, II and III have been accomplished by including the threshold stress intensity parameter Δ K.

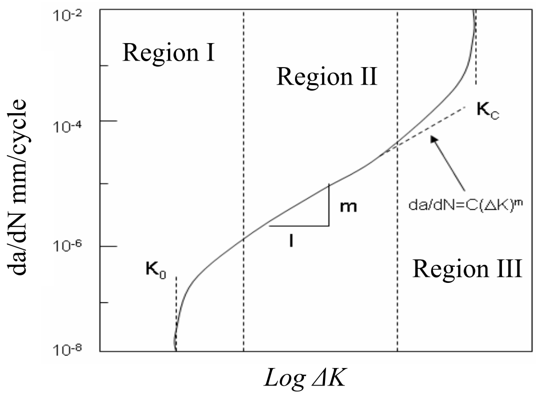

Fig. 1.: Typical *da/dN* versus *ΔK* curve

NASGRO extends the generalized Willenborg *et al.* (1971) model taking into account the reduction of retardation due to underloads and gave the following mathematical equation

$$\frac{da}{dN} = \frac{C\,(\Delta K_{eff})^m}{(1-R_{eff})K_C - \Delta K_{eff}} \qquad (8)$$

$$\Delta K_{eff} = (K_{\max})_{eff} - (\Delta K_{\min})_{eff}$$

$$K_{\max,eff} = K_{\max} - K_{red}$$

$$K_{\min,eff} = \max\left\{\left(K_{\min} - K_{red}\right), 0\right\},$$

$$\text{for } K_{\min} > 0$$

$$= K_{\min} \qquad \text{for } K_{\min} \le 0$$

$$K_{red} = \phi\left[(K_{OL})_{\max}\left[1-\left(\frac{a_n - a_{OL}}{\gamma P_{ol}}\right)\right]^{1/2} - K_{\max}\right]$$

(9)

$$\phi = 2.523\phi_o / (1.0 + 3.5(2.5 - R_U)0.6), R_U < 0.25 \quad (10)$$

$$= 1.0 R_U \geq 0.25$$

$$R_U = \sigma_{UL} / \sigma_{\max,OL}, \qquad \phi_O = 0.2 \quad to \quad 0.8$$

The limitation of this model is the value of ϕ_O is to be obtained from experimentally.

METHODOLOGY

In this application a thin wall tube of aluminum alloy (radius=500 mm and 50 mm in thickness) with circumferential crack under tensile load. The mechanical and fatigue properties of the material used are shown in Table 1.

Table .1: Mechanical and fatigue properties of the material

Yield Stress (MPa) YS	365.422
Ultimate Tensile Strength (MPa) UTS	455
Plane Strain Fracture Toughness $(MPa(m^{1/2}))$ K_1C	36.262
Plane Stress Fracture Toughness $(MPa(m^{1/2}))$ K_1D	72.524
Parte Through Fracture Toughness $(MPa(m^{1/2}))$ K_1E	50.547
Paris Co-efficient $(m/MPa(m^{1/2})^n)$ C	1.5451E-10
Forman Exponent n	3.284
Walker Exponent m	0.3
NASGRO Exponent p	0.5

Components, structures are subjected to quite diverse load histories, their histories may be rather simple and repetitive, at the other extreme, may be completely random. The variable and constant load histories used are shown in Figure 2 and 3 respectively. To account load ranges and mean of the used load history, rain flow accounting method was used. The modeling and simulation of the analysis were done based on Glyphwork codes.

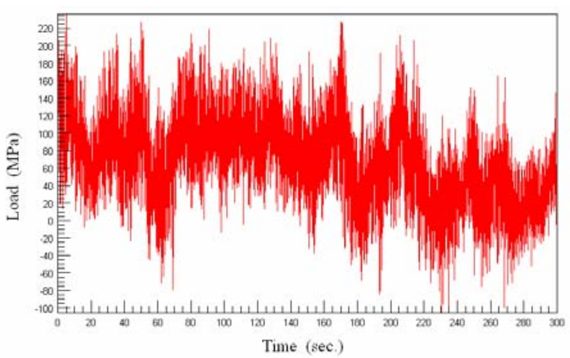

Fig. 2.: Display of Variable Amplitude Loading

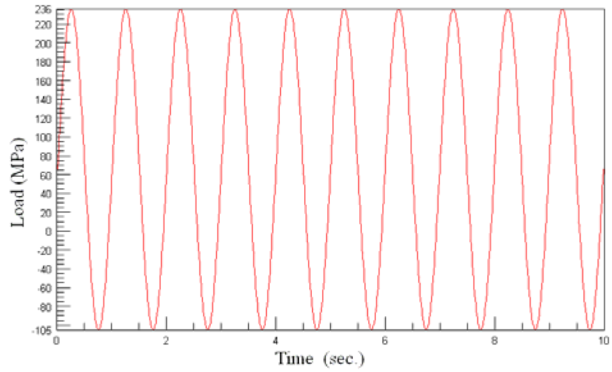

Fig.3.: Display of Constant Amplitude Loading

RESULTS AND DISCUSSION

The effects of the above loading (CAL and VAL) are using the maximum value of VAL as the value of CAL on FCG of the shell was studied and shown in Figure 3. FCG under CAL gives less life than under VAL. CAL account for all cycles as the maximum value, while in VAL only the peaks. Figure 4 also shows the effect of using different FCG models.

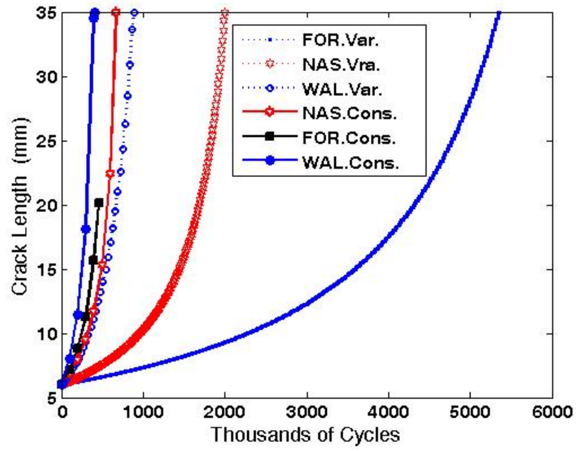

Fig.4.: Fatigue Crack Growth of Thin Wall Tube with CAL and VAL using Different FCG Models

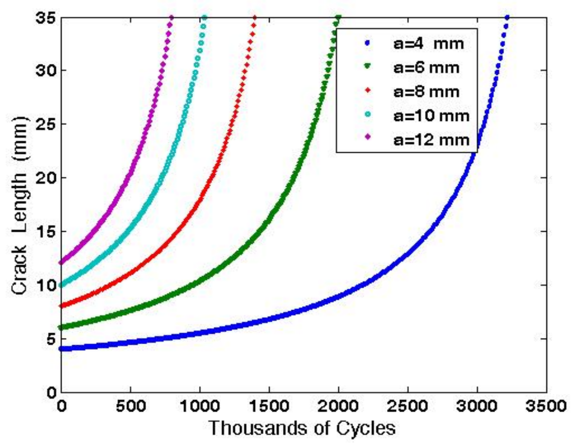

Fig. 5.: Fatigue Crack Growth for Different Initial Crack Length

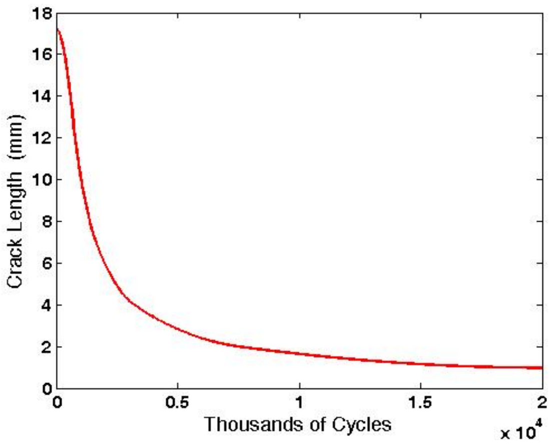

Fig. 6.: Fatigue Life with Different Initial Crack

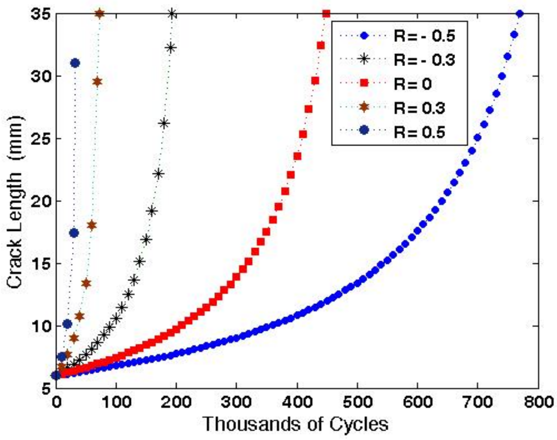

Fig. 7.: Fatigue Crack Growth with Different Stress Ratio

For the models with VAL, Forman model gives longer life and more than NASGRO model, while Walker model gives the lowest life. For CAL, NASGRO model gives the longer life, and Walker model also gives the minimum life. The effect of initial crack length on FCG using NASGRO model was studied and shown in Figure 5. The interactions between them indicate that the higher value of initial crack length tend to the lower number of cycles for crack growth. In the same way, the fatigue life drawn verses the initial crack length and shown Figure 6, which gives a minimum life for larger initial crack length. Most mean stress effects on crack growth have been obtained for only positive stress ratios, i.e., R≥ 0. Figure 7 shows the variation of the number of fatigue loading cycles versus the corresponding crack length for different stress ratios. The Forman model, which used here is a modification of the Paris equation that incorporates mean stress and region III fatigue crack growth behavior. It observed that the FCG affected by different stress ratio, which shown on a-N curve that increasing the stress ratio (which means increasing the mean stress) has tendency to increase the crack growth rates. It should be recognizes that the effect of the stress ratio on fatigue crack growth behavior is strongly material dependent.

CONCLUSIONS

The study of fatigue crack propagation examines how a fatigue crack grows under cyclic load. This topic is the subject of considerable research, mainly dealing with the development of various models to better explain the crack propagation phenomenon. For the modern high performance structures designed for finite service life, fatigue crack growth occurs over a significant portion of the useful life of the structures. Therefore, accurate simulation of crack propagation paths in engineering materials is what designers and engineers are always looking for. A fatigue crack growth of thin walled aluminum alloy with circumferential crack under constant and variable amplitude loading in tension was studied and showed the behavior of FCG with the different load histories. A crack growth calculation procedure based on a three different model Walker, Forman and NASGRO based Glyphwork codes. These models applied in both constant and variable amplitude loading situations gave different behavior under the two types of loading. Walker improved the Paris model by including the effect of stress ratio while, Forman improved the Walker model by suggesting a new model, which is capable of describing region III of the fatigue rate curve and includes the stress ratio effect. NASGRO extended the generalized Willenborg model by taking into account the reduction of retardation due to underloads. Among many factors affecting the FCG, the effect of initial crack and stress ratio are shown. This study indicate that the higher value of initial crack length tend to the lower number of cycles for crack growth, and also showed that increasing the stress ratio (which means increasing the mean stress) has tendency to increase the crack growth rates. It should be recognizes that the effect of the stress ratio on fatigue crack growth behavior is strongly material dependent.

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