

CAN LARGE EDDY SIMULATION (LES) PREDICT LAMINAR TO TURBULENT FLOW TRANSITION?

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ABSTRACT

Four numerical experiments have been conducted using Large Eddy Simulation (LES) to visualize laminar and turbulent incompressible flows over a horizontal finite cylinder. The unsteady velocity fluctuations were studied at a point on the near wake downstream the cylinder. These fluctuations were used, through the fundamental understanding of LES mechanism, as indications of the LES sensitivity in capturing the occurrence of transitional flow. Such proved sensitivity is deemed to depend on the Reynolds number and problem physics. The purpose of this study is to investigate the capability of LES in capturing laminar-to-turbulent transitional flow dynamics. Moreover, the present article describes a generalized methodology to use LES in predicting the spatial and temporal onsets of transitional flow. The significance of this study is pointed out from the fundamental dilemma of predicting transitional flow, and the wide application domain of such phenomena.

Keyword: LES, CFD, Transitional flow, Turbulence model, Laminar to turbulent transition

1. INTRODUCTION

It is a fact that transitional flow is involved in the majority of engineering applications which comprise fluid flow. Such a fact drastically influences the contemporary engineering research concerns and objectives. Engineers, mathematicians, and physicists have been trying to formulate and build proper mathematical and physical models that are able to predict the specific spatial and temporal conditions, at which a specific flow field transits from laminar to turbulent. This mission is, undeniably, quite complicated and ambiguous, especially when it deals with sophisticated physical phenomena, such as those associated with aeronautics and flight engineering. Although most of these applications entail fully turbulent flow, it is quite critical, in some cases, to have the power to predict the very beginning of such turbulence, in terms of time and flow velocity. Turbulence modeling has been a matter of intensive research for the last few decades. These efforts resulted in producing numerous turbulence models, which are

all capable of predicting fully turbulent flow. The effect of laminar-turbulent transition, however, has not been enclosed in such robust turbulent models represented in modern CFD fashion. In the same time the CFD community is trying to develop transition models, substantial efforts are being done to examine the qualification of fully turbulent models to predict transitional flow dynamics. Low-Re models have been used before to capture transition (Jones and Launder 1973, 1984). However, as these models depend on the capability of damping functions to predict both the transition dynamics and the viscous sub layer, an independent simulation of both phenomena cannot be achieved. For instance, it is dubious that models calibrated for viscous sub layer modeling would adequately capture the effects of transition. Other conventional Reynolds averaged Navier Stokes (RANS) procedures cannot be implemented easily to describe transitional flows. This is the consequence of eliminating the effects of linear disturbance growth incorporated in RANS models (Menter *et al.* 2006). RANS models represent the turbulence effects on the mean flow through the modeling of the Reynolds stress term. In contrast, the mathematical foundation of LES makes it a suitable tool for transition prediction. LES foundation adopts the Kolmogorov's self similarity theory, which classifies turbulent eddies to large scale, which are problem dependent, and small scale (i.e. isotropic) universal eddies (Menter *et al.* 2006). Accordingly, LES resolves the large scales directly by solving the filtered conservation equations and models the small scales which are believed to be isotropic and problem independent. Transitional flow occurs due to the development of large eddies, which is a problem dependent phenomenon. From this similarity between the two mechanisms of LES and transitional flow, it is concluded that the first is capable of predicting the latter.

2. LITERATURE REVIEW

Recently, a number of researchers have used LES to study transitional flow in some specific engineering problems such as boundary layer separation and natural convection. However, the majority of these studies were basically concerned about turbulent flow that follows the transition. In addition, there has been no

benchmarking methodology to use LES to predict the transitional flow. Otherwise, each study has suggested a problem-specific technique to provide physical reasoning of LES results that implies the predictability of flow transition. Such demanded methodology should characterize specific flow field parameters, which are mostly indicative of transitional flow, and encapsulate them in LES solution framework to present a generalized method of predicting the laminar-to-turbulent transition. However, these previous researches support the present study by successfully conferring the abovementioned statement on the resemblance between LES mechanism and transitional flow development.

Monokrousos *et al.* (2008) have simulated boundary layer transitional flow over a flat plate subjected to high levels of free stream turbulence using both Direct Numerical Simulation (DNS) and LES. The results of the simulation were used further to control the boundary layer disturbances. Hence delaying the transition process through blowing and suction at the wall, based on the full knowledge of the instantaneous velocity fields. Rodi (2006) has studied laminar-turbulence transition of the boundary layer flow on the blade surfaces in a low pressure turbine cascade with wakes passing periodically through the cascade channel using DNS and LES. Dimas *et al.* (2003) applied LES to capture the sub critical transition to turbulence in the boundary layer attachment line of a swept wing. A dynamic eddy viscosity model was used to parameterize the unresolved scales in the filtered Navier-Stokes equations. This model was expected to provide improvements over the standard Smagorinsky sub grid scale model in predicting the transitional flow dynamics (Hallbaeck *et al.*, 1996). Hickel and Adams (2008) have modeled the laminar to turbulent transition in a zero pressure gradient boundary layer using LES. The work aimed to evaluate the performance of a new implicit sub grid scale model called the adaptive local deconvolution model (ALDM). Such model was developed to overcome the coupling between the truncation error of the numerical discretization and the conventional sub grid scale model. Padilla and Silveria-Neto (2008) captured the thermally induced flow instabilities in natural convection in a horizontal annular cavity using LES for a wide range of Rayleigh number. The dynamic sub grid scale model was used instead of the Smagorinsky model. Abdalla and Yang (2004) have thoroughly investigated the primary and secondary instabilities of a separated boundary layer transition on a flat plate with a blunt leading edge using LES. They also employed the dynamic sub grid scale model. Other successful modeling cases of transitional flow using LES (Michelassi *et al.*, 2002 and Wu *et al.*, 1999)

3. OBJECTIVES AND STRATEGY

This study presents a methodology to predict and monitor the onset of transition to turbulence using LES

based on the temporal velocity fluctuations. Since LES is the only turbulence model, disregarding DNS, that involves the resolving of the large scale turbulent eddies, it is capable to predict the temporal fluctuations of the flow field variables in any specific spatial location of the flow field. Therefore, the commencement of turbulence transition is captured at the corresponding Reynolds number in this specific location as the flow field variables (e.g. velocity field) start to exhibit chaotic fluctuations with respect to time. This cannot be achieved using RANS models since these models do not resolve the turbulent eddies responsible for these temporal fluctuations, instead the effect of these eddies on the mean flow is approximated through modeling the Reynolds stress term (De Comb, 1992; Drikakis and Rider, 2005)

To evidently demonstrate the proposed methodology, further in a quantitative manner, a case study is considered herein. The case study represents a three dimensional viscous flow over a finite horizontal cylinder. Four different Reynolds numbers were considered for the simulation which is conducted using LES. The results of the four simulations are compared to illuminate the proposed methodology

4. LES FORMULATION AND LIMITS

The first results obtained by LES were reported by Deardorff (1970; 1974a; 1974b). In this period, major advances have been added to this method, especially in terms of the models used, the numerical scheme, and the underlying theory. In the following three decades, the substantial progress in digital computers allowed the use of LES in more complex flows. In the present time, LES models exist in the major commercial and open source CFD codes, and are considered valuable simulation tool for both academic and industrial researches.

LES can be thought of as an intermediate technique between DNS and RANS. In the second, the effect of turbulence fluctuations appears in the Reynolds stress term. In order to close the system of equations, this unknown Reynolds stress term is closed using closure approximations and experimentally derived constants. These constants are set using simple flows, for which theoretical solutions or well documented experiments are available. This is the main reason behind the lack of generalization of RANS models. This shortcoming can also be attributed to the fact that these models must represent a wide range of scales. While the small scales have universal characteristics and are more isotropic, the larger scales depend strongly on boundary conditions and are indeed anisotropic. In LES, on the other hand, the large scales are computed, and the small scales are modeled. Since the small scales tend to be isotropic and problem independent, there is a bigger potential to develop universal models for these scales than for RANS equations.

LES relies on resolving most of the turbulent kinetic energy (K) of the flow, while modeling most of the dissipation term (ϵ). The possibility of this separation arises from the fact that K is determined by the large scales of motion and ϵ by the small scales. This big difference in scale between the resolved and the modeled scales allows for even simple models to be sufficient for modeling the small scales. For a reliable LES computation, at least 80 % of the kinetic energy must be resolved as estimated by Pope (2000).

The governing equations employed for LES are obtained by filtering the time-dependent Navier-Stokes equations. The filtering process efficiently filters out eddies with scales smaller than the filter width or grid spacing used in the computations. The resulting equations thus govern the dynamics of large eddies. Filtering the incompressible Navier-Stokes equations, one obtains (the filtered variable is denoted by an over bar):

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\bar{u}_i \bar{u}_j) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} \quad (2)$$

Where τ_{ij} is the subgrid-scale stress defined by:

$$\tau_{ij} \equiv \overline{u_i u_j} - \bar{u}_i \bar{u}_j \quad (3)$$

The sub grid-scale stresses resulting from the filtering operation are unknown, and require modeling. The majority of sub grid-scale models in use today are eddy viscosity models of the following form:

$$\tau_{ij} - \frac{1}{3} \tau_{kk} \delta_{ij} = -2\nu_t \bar{S}_{ij} \quad (4)$$

where ν_t is the sub grid-scale turbulent viscosity, and \bar{S}_{ij} is the rate-of-strain tensor for the resolved scale.

Although a variety of sub grid scale models already exists, the matter of sub grid modeling is a subject of recent debates. For the majority of flows, it has been argued that there is little to gain from refining the sub grid scale models. Spalart (2000) has reported that a LES can be performed without a sub grid scale model at all. Instead, an upwind numerical scheme can be used in order to offset the energy cascade and maintain the smoothness of the solution.

One of the most important challenges in LES is the grid independence study, which involves obtaining nearly identical solutions for flow quantities of interest on successively finer grids. This study is merely used by the CFD community as a verification tool for the numerical simulation. Such a grid independent solution is theoretically impossible to attain in a LES simulation. Away from LES, the grid size controls the truncation error of the solution. Thus, a grid independent solution is reached when the value of this error has reached a minimum. In LES, however, the grid size controls both the truncation error and the amount of scales which will be modeled rather than resolved. Hence, the solution will always vary with

respect to the grid size. Moreover, by refining the grid, more of the spectrum is resolved, thus, adding additional unsteady motion to the solution. Consequently, the time step must be adjusted to be equivalent to the smallest resolved eddy turn over time. Boundary conditions specification is another very challenging aspect of LES. At an inflow, it is usually not sufficient to specify only mean flow quantities as the turbulent state must also be specified. Promising work has been conducted in using recycling methods for equilibrium boundary layers, but for many flows adequately specifying inflow conditions remains an area of research.

5. CASE STUDY PHYSICS

Incompressible flow over a cylinder was intensively investigated both numerically and experimentally in literature. The exceptional feature of this type of flow is the unrelenting dependence of flow pattern on the Reynolds number. Below $Re = 200$, the well known Von Karman vortex street persists to great distances downstream the cylinder. Above this Reynolds number, transitional flow occurs in the wake thereby destroying the periodic vortex wake far downstream. At this flow condition the vortex street also becomes unstable and exhibits three-dimensional disturbances leading to greater irregularity. At $Re = 400$ a further change occurs. Turbulent flow commences close to the separation points on the cylinder. This pattern with laminar boundary-layer separation and a turbulent vortex wake persists until $Re = 3 \times 10^5$. When Re number increases above this value, the laminar boundary layer undergoes transition to turbulence almost immediately after separation. The increased mixing re-energizes the separated flow causing it to reattach as a turbulent boundary layer, thereby forming a separation bubble. The final change in the flow pattern occur at $Re = 3 \times 10^6$ when transition to turbulence occurs at the cylinder wall and the separation bubble disappears (Houghton and Carpenter, 2003)

In the present study we examine four Re values. The first problem at $Re=70$, the flow is fully laminar and the temporal velocity component derivatives equal zero. The results of the first simulation are used in contrast with the results of the fourth simulation, where $Re=7 \times 10^6$, to benchmark the proposed methodology. The second and third simulations are used firstly to further investigate the accuracy and capacity of LES to capture the temporal velocity fluctuations. Secondly, the two cases where $Re=150$ and 400 , were used to confine the transitional flow regime using these fluctuations with respect to laminar and turbulent flow regimes.

6. SOLVER AND MESHING FEATURES

The case study problems were solved using the CFD finite volume solver FLUENT 6.3. The computational domain consists of a horizontal cylinder inside a

rectangular enclosure. A tetrahedral grid comprising 347887 volume cells was generated. The geometry and meshing are illustrated in Fig. 1. Variable size meshing was used to produce smaller cells near the area of concern. Although the hexahedral grid is known to enhance the solution accuracy, the use of tetrahedral grid is justified because a second order central difference scheme was used to discretize both the diffusive and the convective derivatives. Thus, the use of a hexahedral grid is not of actual significance. Moreover, the use of tetrahedral grid allows a bigger number of volume cells in the mesh for the same grid spacing. Hence it allows a larger portion of the energy containing eddies to be resolved using the filtered

Navier-Stokes equations rather than modeled with the sub grid scale model.

An unsteady pressure based solver with second order implicit time discretization was used to enhance the temporal accuracy. Pressure-velocity coupling was achieved using the Semi Implicit Method for Pressure Linked Equations (SIMPLE) algorithm. A time step of one millisecond was used in all the cases which is estimated to be of the order of the large scale eddies turn over time. Convergence has been reached for all governing equations per each time step. A dynamic sub grid scale model was used, which is known to perform better than the standard model in unstable flows ((Damis *et al.*, 2003; Hallbaeck *et al.*, 1996; Padilla and Silveira-Neto, 2008; Abdalla and Yang, 2004).

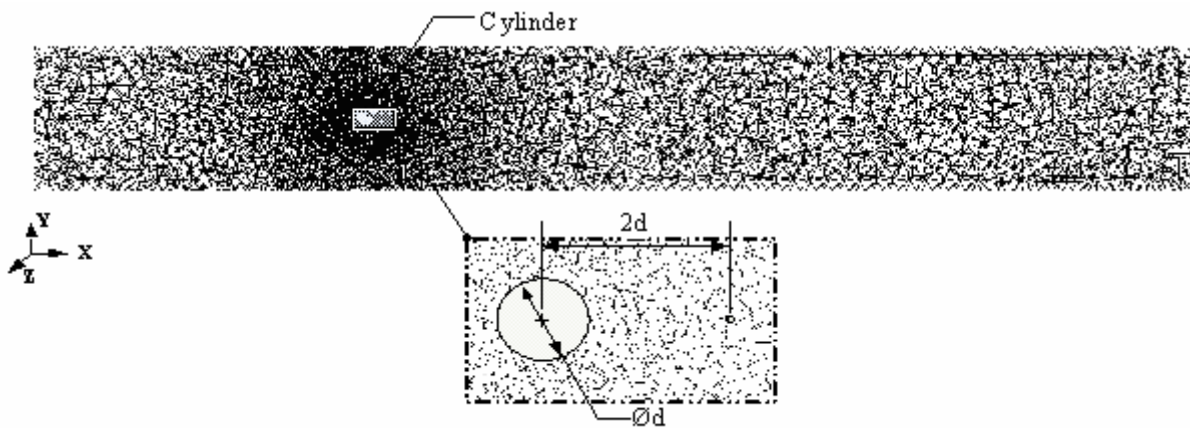
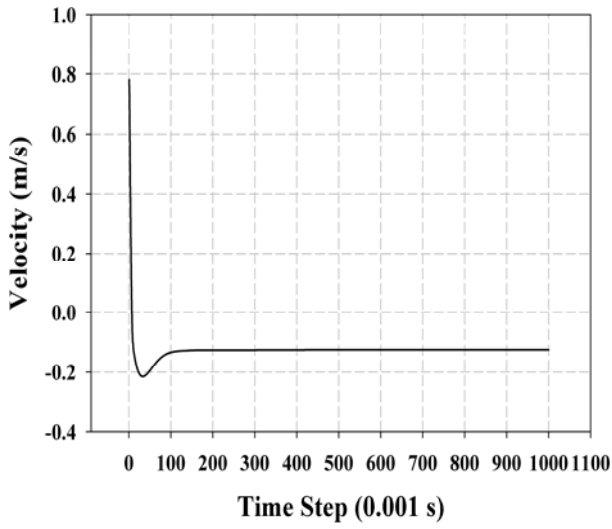


Figure 1. Schematic of the computational domain showing the tetrahedral grid, the cell size growth rate and the location of the issue point

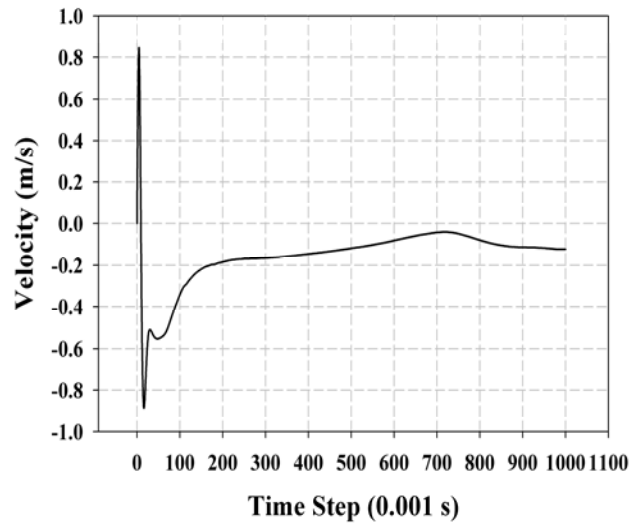
7. RESULTS AND DISCUSSION

Four numerical simulations were performed for at $Re = 70, 150, 400$ and 7×10^6 . The flow history of the three components of velocity at a point in the near wake downstream of the cylinder was recorded at a time step of 0.001 second, with a total flow time of 1 second. The point is located at a distance $2d$ from the cylinder centerline, where d is the cylinder diameter, as in the detailed view in Figure 1. The x , y , and z velocity component fluctuations for the four cases are compared in figures 2, 3 and 4, respectively. The contrast between the laminar and the turbulent flow is obvious (cases (a) and (d) in figures 2, 3 and 4). For the laminar flow case, the fluctuations are present only in the beginning of flow which is due to the flow initialization. After that the flow becomes steady and the velocity components cease to fluctuate with respect to time. At $Re=150$, flow instabilities begin to take place, as indicated by the x , y and z velocity fluctuation as in case (b) of figures 2, 3 and 4, respectively. These fluctuations indicate the sensitivity of LES in capturing the flow dynamics at this critical Re value, where the flow is about to lose its laminarity. At a conceptually similar circumstances, when $Re=400$, LES has also demonstrated a significant sensitivity in capturing the fluctuation of the velocity

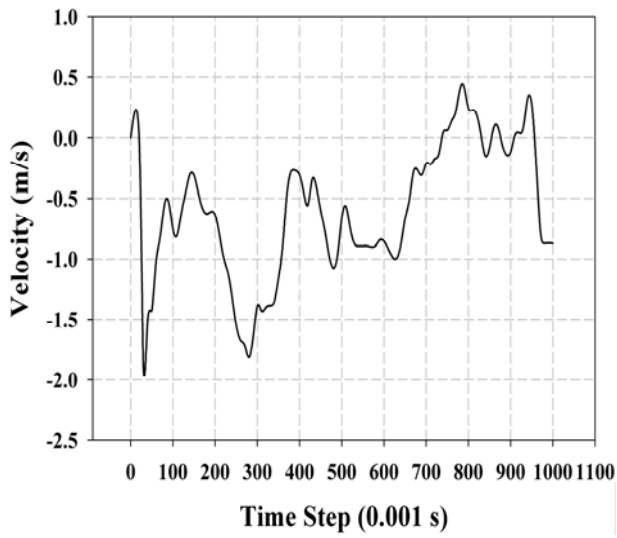
components. At such Reynolds number, the flow enters a turbulence regime, which is well characterized by laminar boundary layer separation and turbulent vortex shedding downstream the cylinder. At $Re=400$, the y -velocity component exhibit semi-periodic fluctuations, as in figure 3-c, which is clearly distinguishable from the fully turbulent chaotic fluctuations shown in figure 3-d. This distinction implies the difference between the two flow regimes; transitional and fully turbulent regimes. In the fully turbulent flow case, the velocity components continue to fluctuate in a completely chaotic style. These fluctuations are caused by the turbulent eddies which are resolved using LES. Undoubtedly, if a RANS model was used to model the turbulent case, these fluctuations would not be apparent. A RANS model would model the effect of these eddies on the mean flow. Nevertheless, the resulting fluctuations would not be obtained, and capturing the transition phase would rely on the values of the turbulence field such the turbulence kinetic energy production or dissipation. This would be accomplished using the damping terms which were originally implied in the turbulence scales equations to resolve the viscous sub layer.



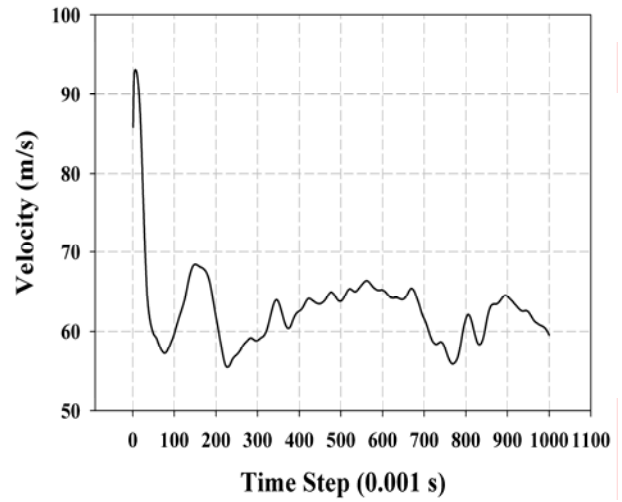
(a)



(b)

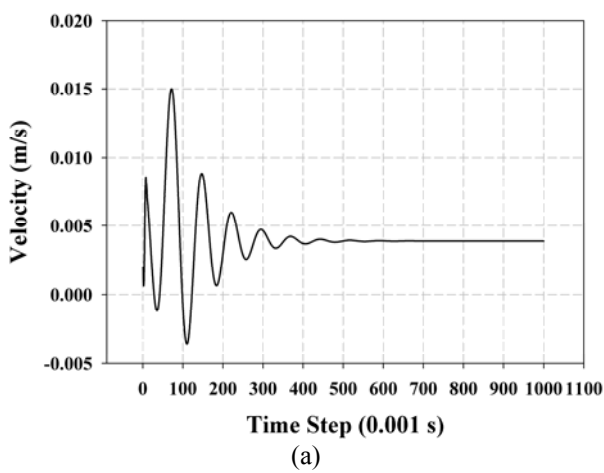


(c)

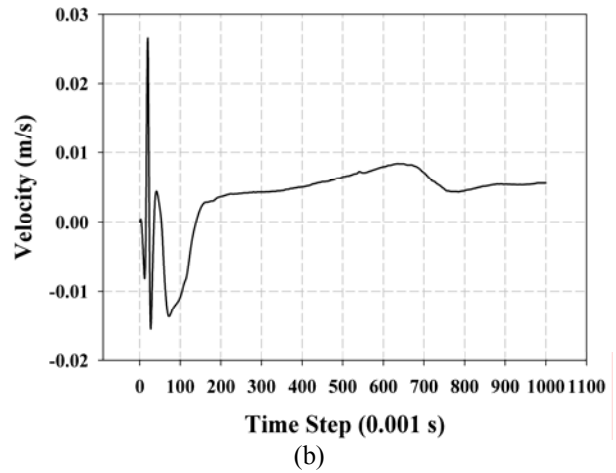


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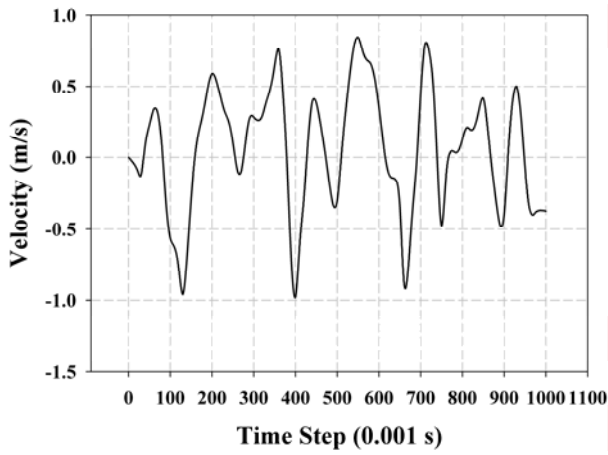
Figure 2. Unsteady x-velocity component of (a) laminar flow (b) $Re = 150$ (c) $Re = 400$ (d) Turbulent flow



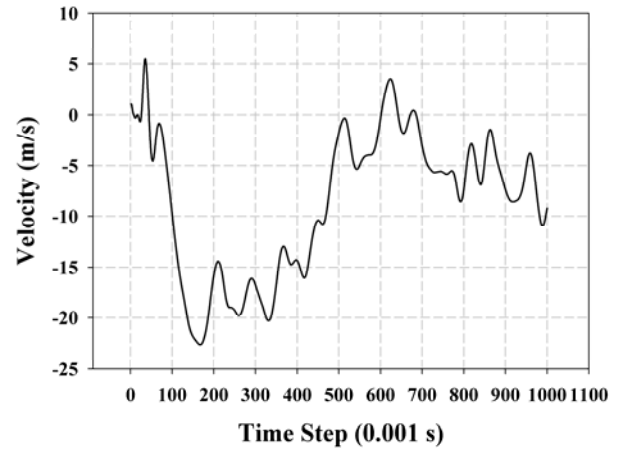
(a)



(b)

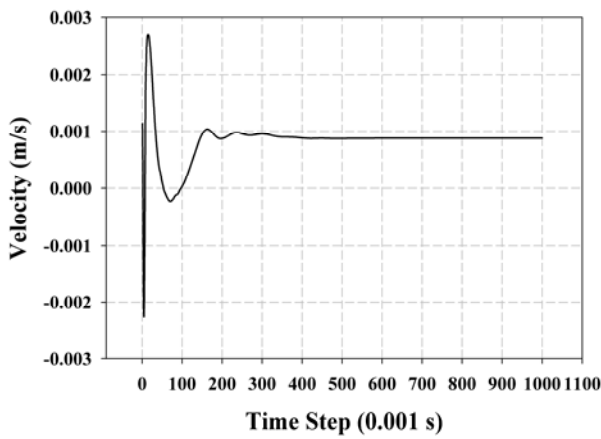


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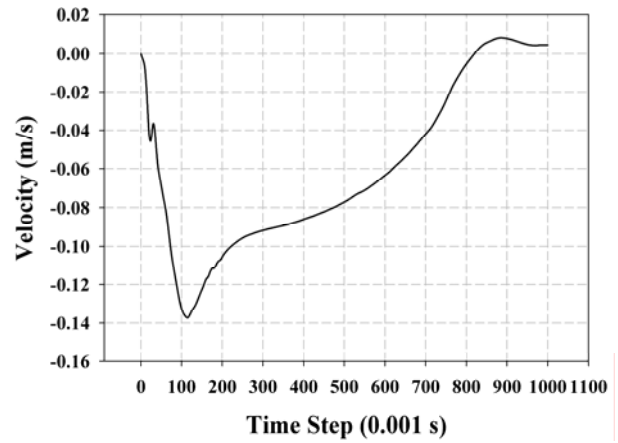


(d)

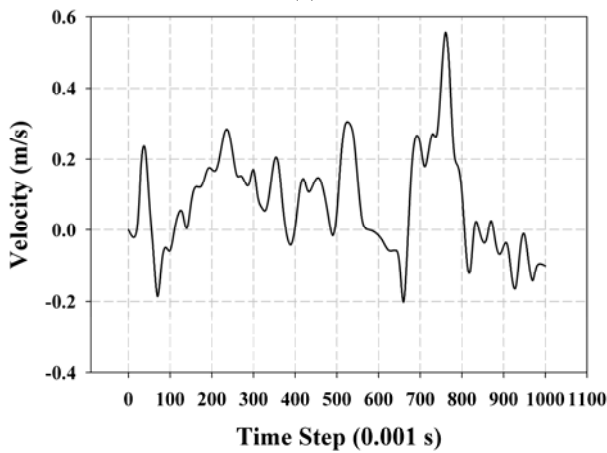
Figure 3. Unsteady y-velocity component of (a) laminar flow (b) $Re = 150$ (c) $Re = 400$ (d) Turbulent flow



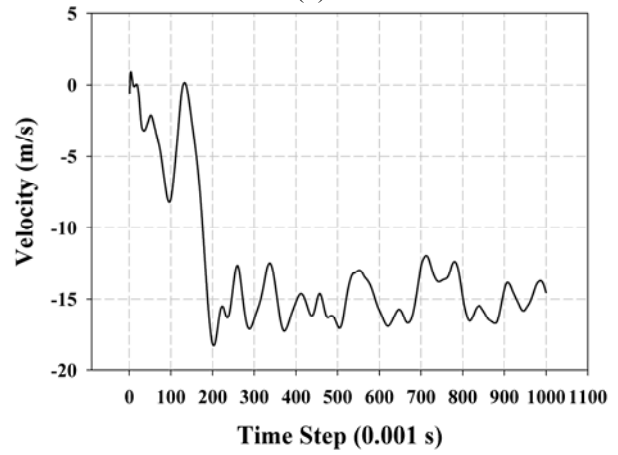
(a)



(b)



(c)



(d)

Figure 4. Unsteady z-velocity component of (a) laminar flow (b) $Re = 150$ (c) $Re = 400$ (d) Turbulent flow

8. Conclusion

The main objective of this study was to examine the capability of LES technique to capture the onset of transitional flow. In order to achieve this objective, four computer experiments have been conducted. The

results of these experiments illuminated the difference between laminar and turbulent flow through the unsteady velocity fluctuations. The investigation of such velocity components focused on a point on the near wake downstream a horizontal cylinder subjected to the four flow regimes. LES was found to be able to

sensitively capture the temporal variation of the three dimensional velocity components at a specific location. Thus, LES can be used with incremental Re number variation to detect the exact Re number, at which transition would occur at a predetermined location. Alternatively, the same methodology can be used, at a specific Re number, to predict the location at which the flow would transit from laminar to turbulent. In addition to such methodology, this study strongly supports the idea that LES contain sufficient qualities to predict transitional flow occurrence and dynamics.

Future Work

Extensive research work is required, in order to fully comprehend the proposed methodology for predicting laminar to turbulent transition using LES. Different flow case studies should be examined using LES within the transitional Reynolds number limits in order to investigate the accuracy of LES in capturing the fluctuations of velocity at different point along the flow field.

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