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HEAT TRANSFER ANALYSIS OF POROUS MEDIUM IN A CONICAL CYLINDER WITH VARIABLE WALL TEMPERATURE

N.J. Salman Ahmed[1], Z.A. Zainal[1], Irfan Anjum Badruddin[2] and M.T. Khalid Hussain[1]
[1]School of Mechanical Engineering, Universiti Sains Malaysia, Pinang, Malaysia
[2]Dept of Mechanical Engineering University Malaya, Kuala Lumpur Malaysia.
Email : Irfan_magami@Rediffmail.com

ABSTRACT

The study of heat and mass transfer in porous media has received special attention from the researchers in recent years, which is witnessed by the large number of publications in this area. The versatility of this field is such that it covers large number of practical aspects of human life such as petrochemical refineries, thermal insulation of buildings, solidification of alloys, drying processes, nuclear waste disposal etc. Even though a number of articles are published on this topic, still many aspects remain unexplored. In the present study, fluid flow behavior in the complex geometry such as conical cylinder filled with saturated porous medium is analyzed. The inner surface of the conical cylinder is assumed to have variable wall temperature. The governing partial differential equations are non-dimensionalised by suitable non dimensional parameters. These equations are then solved by finite element method. The coupled partial differential equations are converted to the matrix equation by Galerkins method. The domain under consideration is meshed by using 3 nodded triangular elements. The variation of Nusselt number corresponding to the various parameters such as cone angle, aspect ratio of the cylinder is presented. It is seen that the effect of cone angle on the heat transfer rate is significant.

Keywords: Porous media, conical cylinder, Natural convection, Radiation, Cone angle, FEM.

NOMENCLATURE

c_p Specific heat (J/kg-^{o}C)
g Gravitational acceleration (m/s^2)
H Height of the Annular cone (m)
k Thermal conductivity (W/m-^{o}C)
K Permeability of porous media (m^2)
L = $r_o - r_i$ (m)
$\overline{Nu}$ Average Nusselt number
q_r Radiation flux (W/m^2)
r, z Cylindrical co-ordinates (m)
$\bar{r}$, $\bar{z}$ Non dimensional co-ordinates
R_r Radius ratio
Ra Rayleigh number
R_d Radiation parameter
$T, \overline{T}$ Dimensional (^{o} C) and non dimensional temperature respectively
u,w Velocity in r and z directions (m/s)

Greek Symbols
α Thermal diffusivity (m^2/s)
β Coefficient of thermal expansion (1/^{o}C)
β_R Absorption coefficient (1/m)
λ Power law exponent
θ Cone angle
ρ Density (kg/m^3)
v Coefficient of kinematic viscosity
σ Stephan Boltzmann constant
ψ , $\overline{\psi}$ Dimensional and nor dimensional Stream function respectively

Subscripts
h Hot
∞ Conditions at outer radius
i Inner
o Outer

INTRODUCTION

The Natural convective heat transfer in a saturated porous media has attracted the researchers' attention over the years. It has various practical applications in the geophysical sciences, petroleum refinery, heat exchangers, thermal insulation of the buildings, cooling of nuclear reactors etc. The different aspects of porous medium have been well documented in the literature. The books by Nield and Bejan (1999); Vafai (2000a); Vafai, (2000b), Pop and Ingham (2001) are excellent source of information in this particular area. Prasad (1986) has carried out experimental investigations of natural convection in a vertical annulus filled with saturated porous medium. Yih (1999b) has explained the effect of radiation on natural convection over a vertical cylinder using finite difference method. Yih (1998a) also considered a vertical cone embedded in a porous medium to study the boundary layer analysis for uniform lateral mass flux effect on natural convection of non-Newtonian power-law fluids. Nath and Stayamurthy (11985) have investigated free convection heat transfer in an annular cylinder filled with porous medium by employing the finite difference method. Prasad and Kulacki (1984) have analyzed the natural convection in a vertical porous annulus. They found that the local rate of heat transfer is much higher near the top edge of the cold wall. Ching-Yang Cheng (2000) has used an integral approach to investigate heat and mass transfer by natural convection from truncated cone in porous media. Murthy and Singh (2000) have explained the thermal dispersion effects over a cone with the help of similarity solution. The analytical solutions for MHD fully developed upward or downward natural-convection velocity and temperature profiles for four boundary conditions in an open-ended vertical concentric porous annuli has been presented by A1-Nimr (1995). Chamkha (1996) has considered the steady, laminar, free convection flow along a vertical cone and a wedge immersed in an electrically conducting fluid-saturated porous medium in the presence of a transverse magnetic field. The effect of thermal radiation on the non-Darcy natural convection flow over a vertical cone and wedge embedded in a porous medium with variable viscosity and wall mass flux was analyzed by Al-Harbi (2005). In the present paper, the natural convection in a conical cylinder with its inner surface maintained at variable temperature is analyzed.

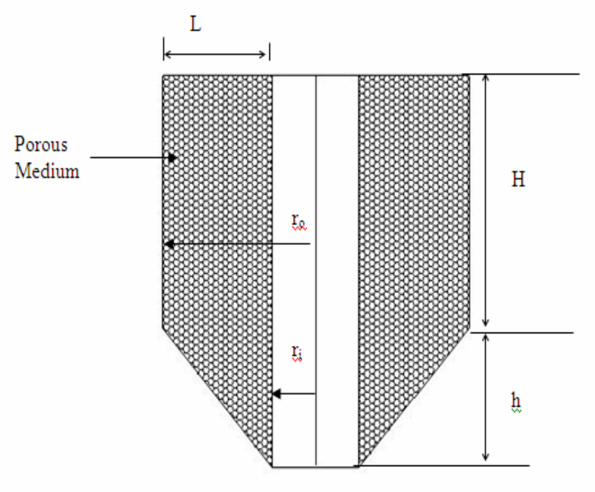

Figure 1: Conical Cylinder filled with porous medium

To the best of author's knowledge, the heat transfer in a conical cylinder fixed with porous medium with variable wall temperature has not been reported thus far.

In this study, porous medium fixed within conical cylinder as shown in the figure 1 is considered. Let r_i and r_o be the inner and outer radius of cylinder respectively. The r axis and the z axis represent the radial and vertical directions respectively. The inner surface of the conical cylinder is maintained at variable temperature, whereas the outer surface is maintained at the ambient temperature. The horizontal surface is assumed to be adiabatic. In the present investigation following assumptions have been considered.

- The convective fluid and porous media are in local thermal equilibrium.
- The properties of the fluid and the porous media are constant.

The fluid is assumed to be gray emitting and absorbing but non-scattering. The governing equations for the heat and fluid flow inside the porous medium can be given as

$$\frac{\partial(ru)}{\partial r}+\frac{\partial(rw)}{\partial z}=0 \quad (1)$$

$$\frac{\partial w}{\partial r}-\frac{\partial u}{\partial z}=\frac{gK\beta}{\nu}\frac{\partial T}{\partial r} \quad (2)$$

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right)-\frac{1}{\rho c_p}\left(\frac{1}{r}\frac{\partial}{\partial r}(rq_r)+\frac{\partial q_r}{\partial z}\right) \quad (3)$$

The continuity equation (1) can be satisfied by introducing the stream function ψ as:

$$u=-\frac{1}{r}\frac{\partial\psi}{\partial z} \quad , \quad w=\frac{1}{r}\frac{\partial\psi}{\partial r} \quad (4)$$

Applying the boundary conditions:

at $r = r_i$, $T = T_h$, $u = 0$ (5a)

at $r = r_o$, $T = T_\infty$, $u = 0$ (5b)

Rosseland approximation for radiation can be given as:

$$q_r=-\frac{4\sigma}{3\beta_R}\frac{\partial T^4}{\partial r} \quad (6)$$

Taylor's series expansion [15] of T^4 about T_∞ , and neglecting higher order terms results:

$$T^4 \approx 4TT^3{}_\infty - 3T^4{}_\infty \quad (7)$$

The following non-dimensional variables are used:

$$\bar{r}=\frac{r}{L} \quad , \quad \bar{z}=\frac{z}{L} , \quad \bar{\psi}=\frac{\psi}{\alpha L} ,$$

$$\bar{T}=\frac{(T-T_\infty)}{(T_w-T_\infty)} , R_d=\frac{4\sigma T_\infty^{\,3}}{\beta_R k} , Ra=\frac{g\beta_T\Delta TKL}{\nu\alpha} \quad (8)$$

After substituting the non-dimensional parameters, equations (2) and (3) take the form:

$$\frac{\partial^2\bar{\psi}}{\partial\bar{z}^2}+\bar{r}\frac{\partial}{\partial\bar{r}}\left(\frac{1}{\bar{r}}\frac{\partial\bar{\psi}}{\partial\bar{r}}\right)=\bar{r}Ra\frac{\partial\bar{T}}{\partial\bar{r}} \quad (9)$$

$$\frac{1}{\bar{r}}\left[\frac{\partial\bar{\psi}}{\partial\bar{r}}\frac{\partial\bar{T}}{\partial\bar{z}}-\frac{\partial\bar{\psi}}{\partial\bar{z}}\frac{\partial\bar{T}}{\partial\bar{r}}\right]=\left(1+\frac{4R_d}{3}\right)\left(\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\frac{\partial\bar{T}}{\partial\bar{r}}\right)+\frac{\partial^2\bar{T}}{\partial\bar{z}^2}\right)$$

(10)

The corresponding boundary conditions are

at $r=\bar{r}_i$, $\bar{T}=1$, $\bar{\psi}=0$ (11a)

at $r=\bar{r}_o$, $\bar{T}=0$, $\bar{\psi}=0$ (11b)

NUMERICAL METHOD:

The heat transfer behavior is predicted by solving the above coupled partial differential equations. Finite element method is used to get the solution of the above coupled partial differential equation. A simple 3 noded triangular element is used to descritise the porous medium. The solution variables $\overline{T}$ and $\overline{\psi}$ vary linearly inside the element and can be expressed as:

$$\overline{T} = N_1\overline{T}_1 + N_2\overline{T}_2 + N_3\overline{T}_3 \quad (12a)$$

$$\overline{\psi} = N_1\overline{\psi}_1 + N_2\overline{\psi}_2 + N_3\overline{\psi}_3 \quad (12b)$$

where N_1, N_2, N_3 are shape functions.

The coupled matrix equation are obtained by solving the above equation by Galerkins method. These coupled matrix equations are assembled for an element to get the global matrix equation for the whole domain. This global matrix equation is solved iteratively to obtain $\overline{T}$ and $\overline{\psi}$. The difference between the two iterations are adjusted to 10^{-5} and 10^{-9} for $\overline{T}$ and $\overline{\psi}$ respectively to get the accurate results. The results are presented in terms of average Nusselt number at the hot surface of the conical cylinder, with respect to various parameters such as the angle of conical cylinder (θ), radius ratio $(r_o - r_i)/r_i$, radiation parameter and Rayleigh number.

The average Nusselt number may be defined as:

$$\overline{Nu} = -\frac{\int_0^{\overline{z}} \left(1 + \frac{4R_d}{3}\right)\frac{\partial \overline{T}}{\partial \overline{r}}\bigg|_{\overline{r} = \overline{r}_i i}}{\overline{z}} \quad (13)$$

The present methodology is verified by simplifying the geometry to vertical annulus and then comparing the results with those of earlier published data. The comparison is shown in table 1. It can be seen that the present method has good accuracy in predicting the Nusselt number.

Table 1 $\overline{Nu}$ comparison to validate present method

Aspect ratio	Prasad and Kulacki [10]	Rajamani et al. [17]	Present
3	3.70	3.868	3.8038
5	3.00	3.025	3.0638
8	2.35	2.403	2.3949

RESULTS AND DISCUSSION

Figure 2 describes the values of the average Nusselt number plotted against the power law exponent λ, for different values of angle of conical cylinder. From the figure, it is clear that the average Nusselt number decreases with the increase in the power law exponent λ for different values of cone angle. For $\lambda = 0$ the process becomes special case of isothermal wall temperature. At the value $\lambda = 0.5$ and beyond the subsequent decrease in heat transfer rate with increase in λ reduces.

Similarly the figure 3 shows the variation of average Nusselt number with respect to different values of power law exponent λ, for 3 values of the aspect ratio i e A_r=0.5,1 and 3. It is seen that the value of average Nusselt number for higher aspect ratio ie A_r=3 decreases significantly with respect to λ as compared to the lower values of aspect ratio. Thus for higher values of aspect ratio the variation in the heat transfer rate is more.

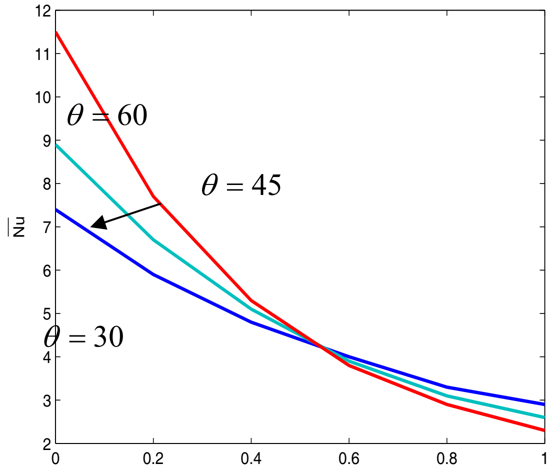

Figure 2 Average Nusselt number Vs λ

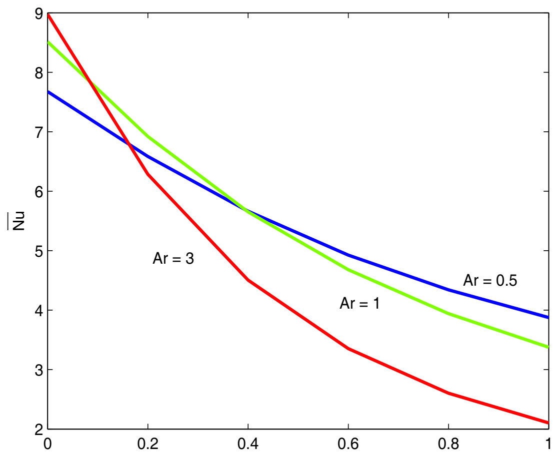

Figure 3 Average Nusselt number Vs λ

Figure 4 reflects the heat transfer rate when radius ratio is varied with respect to λ. The different values of average Nusselt number are plotted against power law coefficient values λ, for three values of radius ratio ie Rr=0.5,1 and 2. It is clear from figure that the average

Nusselt number decreases with increase in the power law coefficient values λ. Figure 5 shows the variation of Nusselt number against the power law coefficient values for 3 different values of radiation i.e. for Rd=0,1 and 2. From figure it is clear that for lower values of radiations such as 0 and 1 the heat transfer rate is much less where as for radiation value 3 the rate of heat transfer increases significantly.

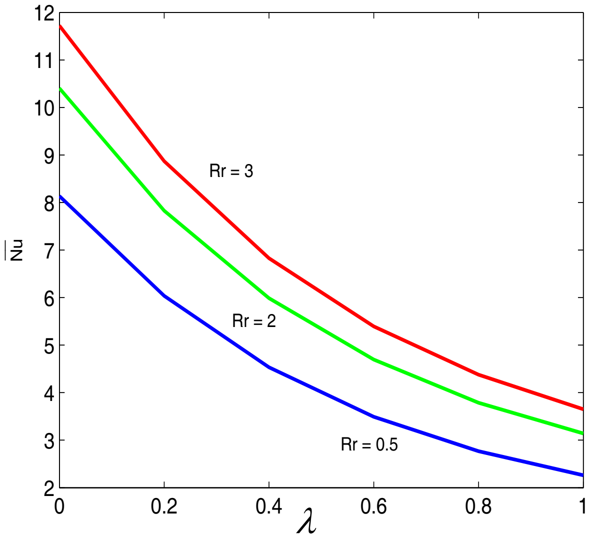

Figure 4 Average Nusselt number Vs λ

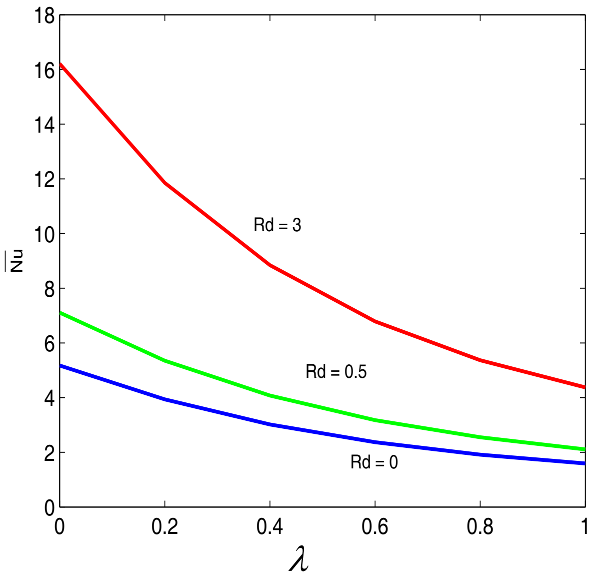

Figure 5 Average Nusselt number Vs λ

Figure 6 shows isotherms and streamlines, for different values of power law exponent i.e. $\lambda = 0, 0.33\ and\ 1$.

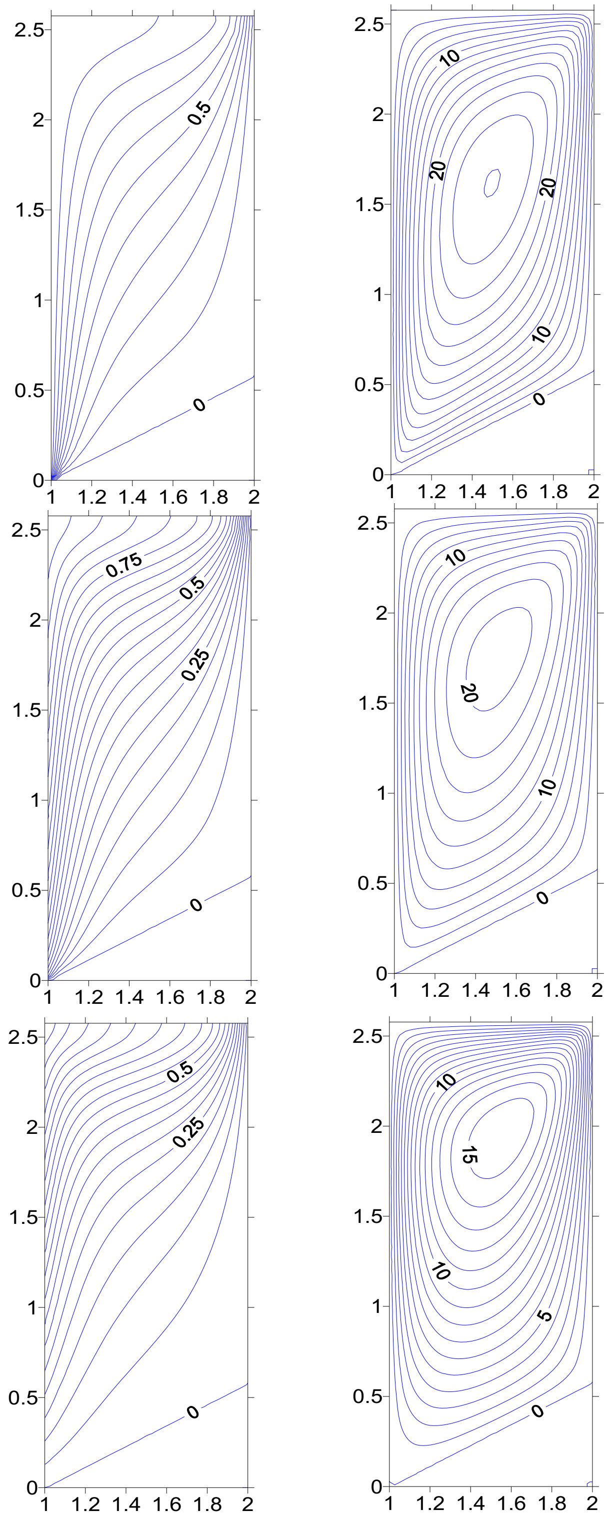

Figure 6 Isotherms and stream lines for $\lambda = 0,\ 0.33\ \&\ 1$ at

$$\theta = 45^0, Ra = 200, R_d = 1, R_r = 1$$

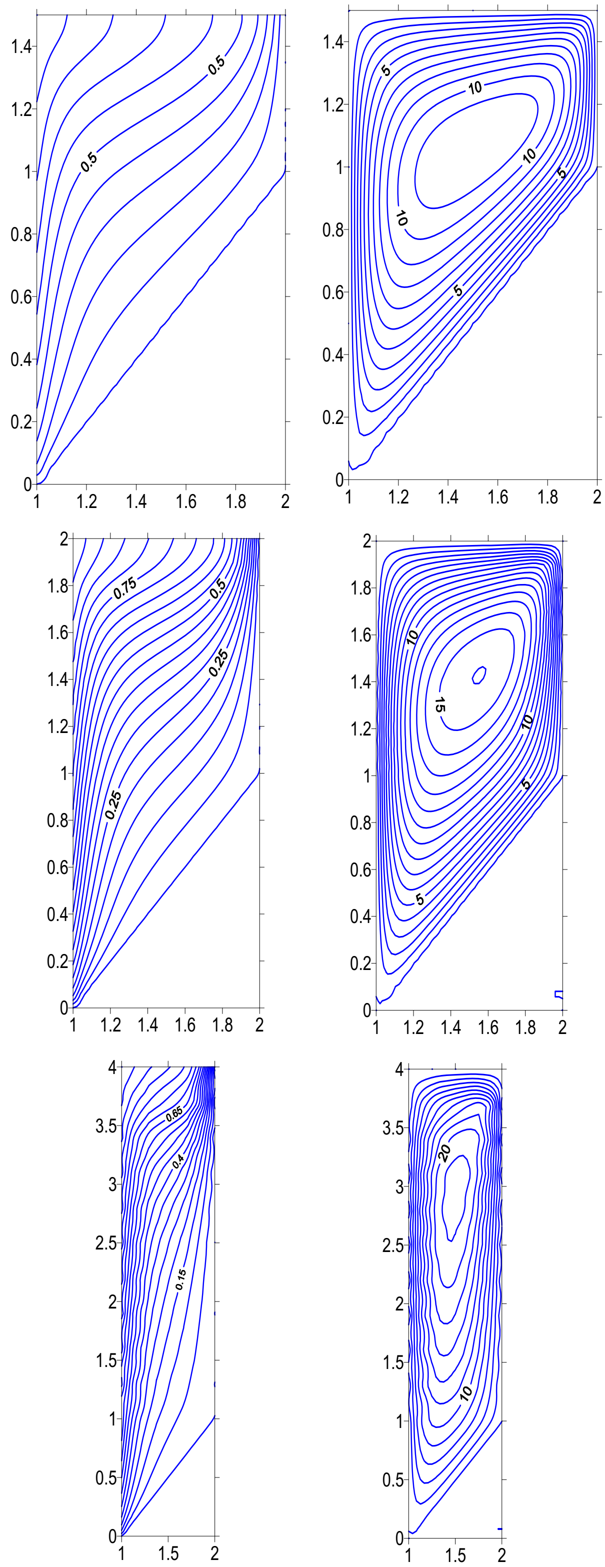

Figure 7 Isotherms and Streamlines for
$A_r = 0.5$, 1 and 3 at
$\lambda = 0.5, R_d = 1, R_r = 1, \ \theta = 45^o$ & $Ra = 200$

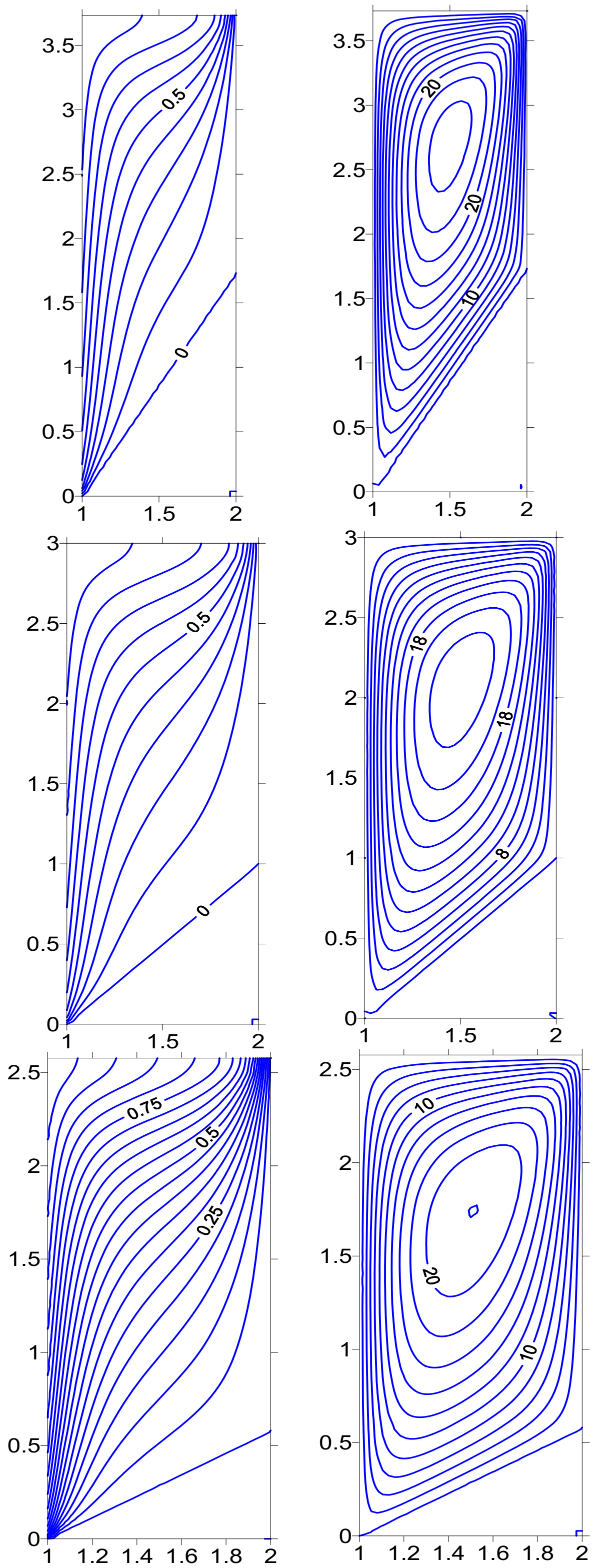

Figure 8 Isotherms and Streamlines for
$\theta = 30^0, 45^0$ *and* 60^0 $Ra = 200, R_d = 1 R_r = 1 \lambda = 0.25$

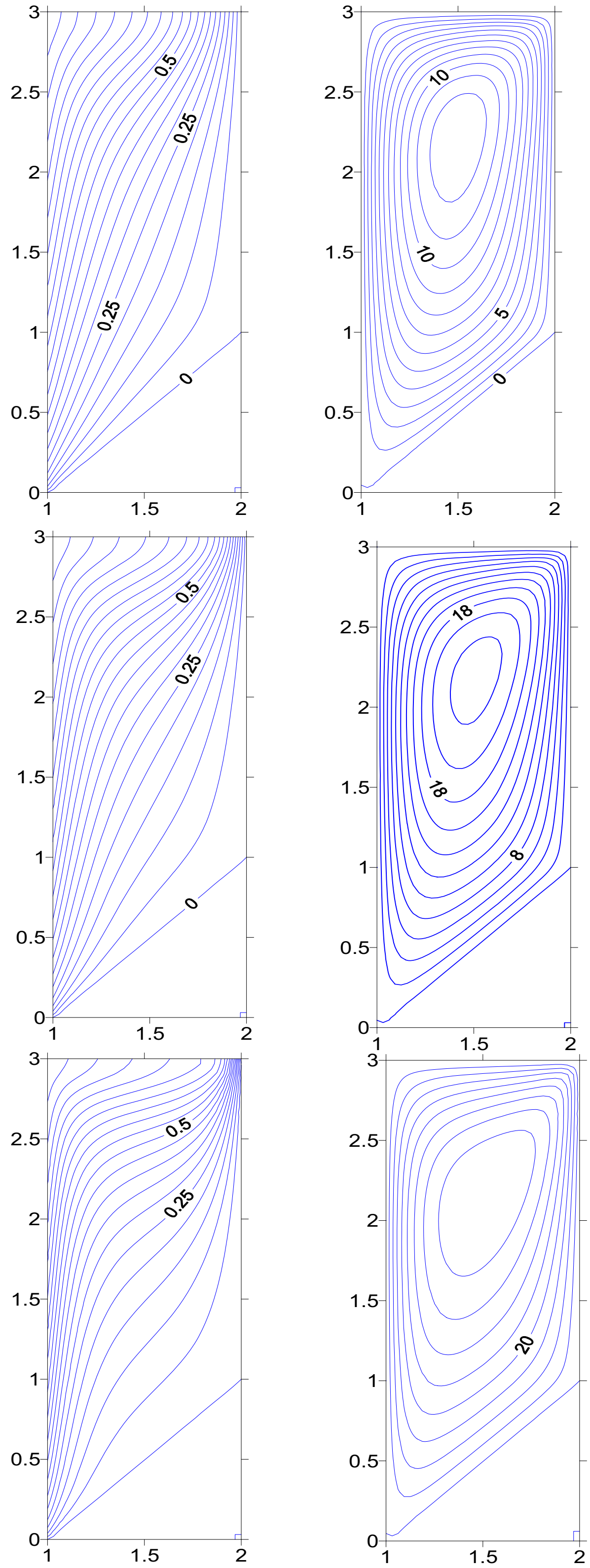

Figure 9 Isotherms and Streamlines for
Ra=100,200,500,Rr=1, $\lambda = 0.5$,Rd=2 $\theta = 45$ Ar=2

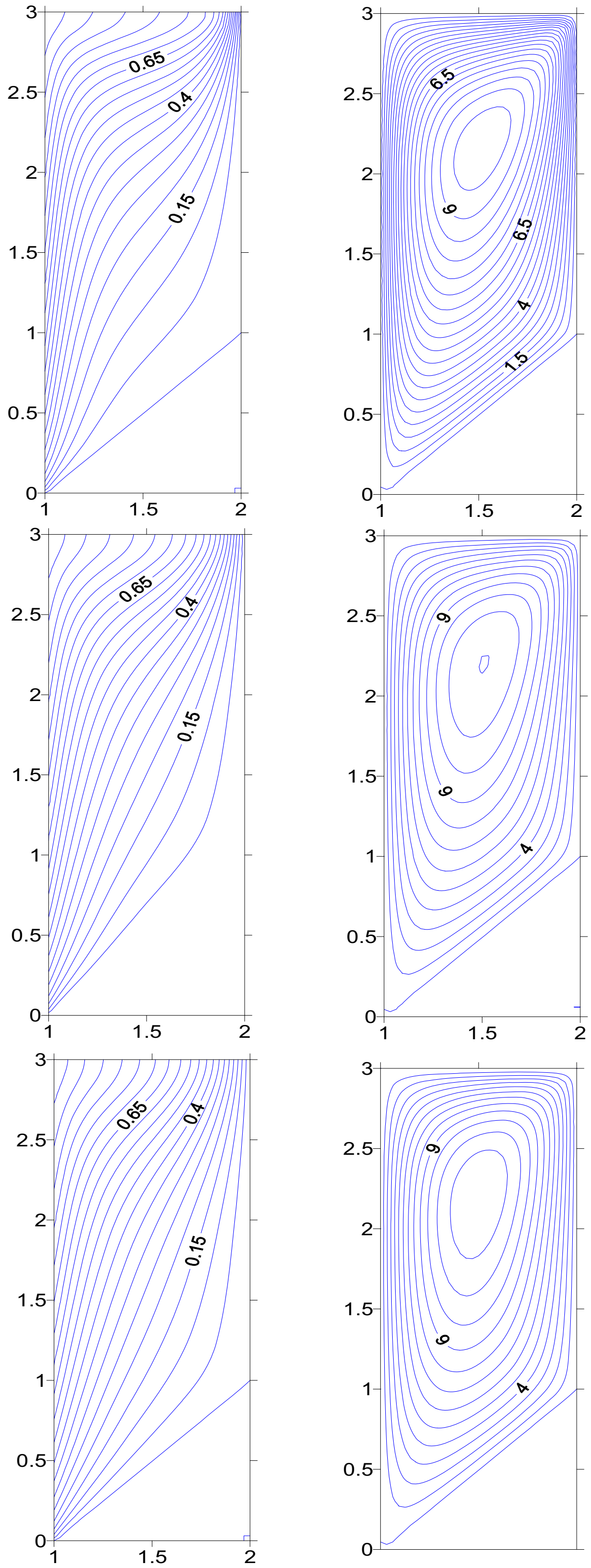

Figure 10 Isotherms and Streamlines for
Rd=0,1,2, Ra=100, Rr=1, $\lambda = 0.5$, Ar=2

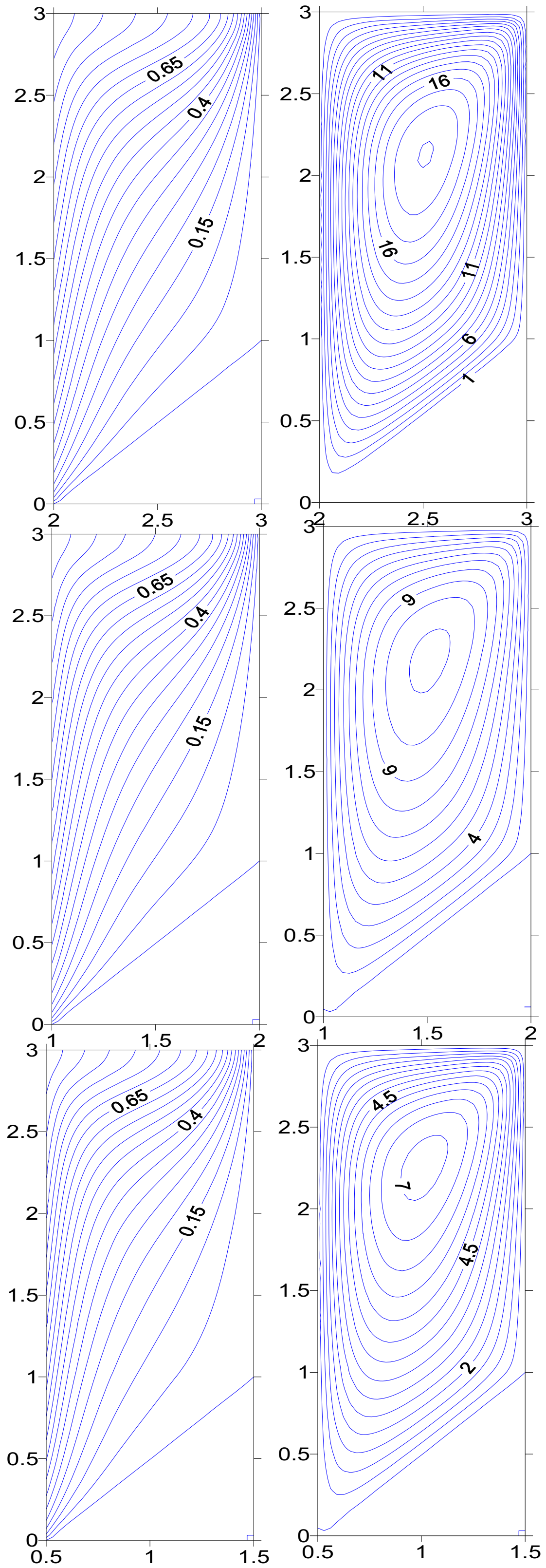

Figure 11 Isotherms and Streamlines for Rr=0.5,1,2, Ra=100, Rd=0.51, $\lambda = 0.5$, Ar=2 $\theta = 45$

The isotherms and streamlines show that when λ is varied from 0 to 1, the heat transfer is predominant at the upper portion of the conical cylinder. Similarly isotherms and streamlines are plotted for different values of aspect ratio of cylinder in figure 7. It is seen that the isotherms move towards the hot surface as the aspect ratio increases. Figure 8 shows the isotherms and streamlines for different values of cone angle. The isotherms move towards the hot surface with increase in cone angle for the parameters $R_d = 1 R_r = 1 \lambda = 0.25$. The fluid flow cell turns more circular when cone angle increases as shown in streamlines of figure 6. Figures 9, 10 and 11 shows the isotherms and streamlines for various values of Rayleigh number, radiation parameter and radius ratio respectively. The isotherms tend to accumulate near the hot surface as the Rayleigh number increasers thus increasing the heat transfer rate from the hot surface. The streamlines for lower Rayleigh number exhibit smooth and more orderly flow. The fluid flow direction tends to be vertical as the radiation parameter increases.

CONCLUSION

The present study explains the numerical investigation of natural convection in conical cylinder fixed with porous medium. The inner surface of the conical cylinder is maintained at variable temperature. The governing partial differential equations are solved to get the matrix equation. Finite element Method is adopted to solve these equations. It is found that the average Nusselt number decreases sharply for decrease in cone angle. It is also found that the aspect ratio of geometry influence significantly the rate of heat transfer.

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