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EVALUATION OF VARIANCE MISMATCH FOR SERIAL TURBO CODES WITH TALWAR PENALTY FUNCTION UNDER INTERFERENCE OF IMPULSIVE NOISE

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Abstract

An investigation has been made to find out the impact of channel variance mismatch to the performance serial turbo codes (SCCC) equipping with talwar penalty function over impulsive channels. The impulsive noise channel is blamed to be the culprit of signal impairment and causes severe performance degradation to modern SCCC which traditionally has been designed to perform optimally in additive white Gaussian noise (AWGN) channel. Undoubtedly, it is a major limiting factors for systems such as Powerline Communication Channel (PLC), digital subscriber line (DSL) and digital TV. SCCC have been proven to approach Shannon Capacity bound in Gaussian channel. The constituent decoders are implemented using the log-MAP and MAX-LOG-MAP algorithms. It is shown that serial turbo decoders with talwar penalty function that are implemented with the LOG-MAP algorithm are sensitive to varying channel variance, where underestimation and overestimation are almost equally detrimental over impulsive noise channel. On the other hand, the performance of the MAX-LOG-MAP based serial turbo with decoders with talwar penalty function exhibit complete independence with respect to errors in the variance estimate in impulsive channel. The study also found that using talwar penalty function can significantly improve decoding capabilities of SCCC in impulse noise channel under varying channel variance environment. Therefore Shannon predicted excellent performance of turbo codes can be perfectly obtained from such impulsive channel.

Keywords: Serial turbo codes, Variance mismatch, Talwar penalty function, Log-MAP algorithm, Max-log-MAP algorithm, Impulsive noise.

Nomenclatures

A	Impulsive Index
$A_{k-1}(`s)$	MAX LOG MAP forward recursion calculation
a	Fading amplitude
$B_k(s)$	MAX LOG MAP backward recursion calculation
E_b	Transmitted energy per bit
L_c	Channel reliability value
$L(u_k)$	A priori LLR
$L(u_k \mid y)$	A posteriori log likelihood ratio (LLR)
$P(u_k = \pm 1 \mid y)$	A posteriori probability (APP) of k data bits
$p(y_k \mid c_k)$	Conditional Probability.
$p_A(z)$	Probability density function (pdf)

Greek Symbols

$\alpha_{k-1}(`s)$	LOG-MAP Forward recursion calculation
$\beta_k(s)$	LOG-MAP Backward recursion calculation
Γ	Gauss-to-Impulse noise Power Ratio
$\Gamma_k(`s, s)$	MAX LOG MAP Branch transition probability
$\gamma_k(`s, s)$	LOG-MAP branch transition probability
σ^2	Noise variance
ρ_{talwar}	Talwar Penalty Function

Abbriviations

APP	A posteriori probability
AWGN	Additive white Gaussian noise
DSL	Digital subscriber line
MAP	Maximum a posterio
PCCC	Parallel turbo codes
PDF	Probability density function
PLC	Powerline communication channel
RSC	Recursive systematic convolutional
SCCC	Serial turbo codes
SNR	Signal-to-noise ratio

1. Introduction

Since their invention in 1993 [1], turbo codes have been widely studied and adopted for next-generation high-speed wireless data services by standards organisations [2]. Depending on the form of concatenation, turbo codes can be classified as either parallel (PCCC) or serial turbo codes (SCCC). The original turbo codes proposed in [1] are called parallel turbo codes (PCCC) because the encoder comprises a parallel concatenation of two convolutional encoders linked by an interleaver, and the decoder involves iterative parallel decoding between two constituent convolutional decoders. Likewise, if the two constituent encoders and decoders are cascaded, we obtain serial turbo codes (SCCC). In fact, SCCC have been shown to outperform PCCC due to higher interleaving gain [3,4].

The maximum a posterior (MAP) or the log-MAP algorithm [5] is the most commonly used iterative decoding algorithm for parallel and serial turbo codes. Deployment of MAP or log-MAP constituent decoders requires knowledge of the channel reliability value L_c, which depends on the signal-to-noise ratio (SNR). It is usually assumed that this side information can be obtained without much difficulty and thus the decoder can operate reliably. Problems arise when the channel varies with time, requiring a continuous update in the decoder. In practical systems, this requisite knowledge needs to be estimated from the noisy channel output particularly from impulsive noise channel, while in some communication applications it may not be feasible to incorporate a channel estimator at all. Variance mismatch is a situation where channel variance is unknown to the receiver and the estimation of the channel variance is difficult to obtain from transmission line. As a result, it is important to study the impact of variance mismatch on their performance and to seek alternative algorithms that are simpler to implement and robust to channel mismatch.

In [6,7], the authors independently studied the performance of turbo codes under variance mismatch in Additive White Gaussian Noise (AWGN) Channel. The impact of variance mismatch to SCCC under AWGN has been studied [8]. The author found that SCCC is more sensitive to variance mismatch compared to PCCC. The general conclusion is that overestimation of SNR is less detrimental than underestimation, tolerating a mismatch of several decibels without significant degradation. Traditionally, PCCC and SCCC are designed specifically to function in AWGN channel. In the presence of impulsive noise, these decoders fail to achieve the desired performance as shown in [1]. A novel technique was introduced in [9] to enable SCCC to function in impulsive and non-Gaussian channel. Talwar penalty function was proposed [9] to SCCC to increase the robustness of decoding procedure and guard against outliers arising from impulsive channel. Remarkable decoding performance was reported in the letter. In this journal, we study the variance mismatch of serial turbo codes with talwar penalty function in impulsive channel.

2. Soft-Input-Soft-Output (SISO) Decoding

We consider a serial turbo encoder consisting of two component encoders concatenated in series and separated by an interleaver as shown in Fig. 1(a). The inner encoder is required to be a recursive systematic convolutional (RSC) code, whereas the outer encoder needs not be. At the receiving end, Fig. 1(b), iterative decoding procedure involves the use of two component decoders that exchange probabilistic information cooperatively and iteratively to estimate the a posteriori log-likelihood ratio (APP-LLR)

$$L(u_k \mid y) = \ln\left(\frac{P(u_k = +1 \mid y)}{P(u_k = -1 \mid y)}\right) \tag{1}$$

where $P(u_k = \pm 1 \mid y)$ is the APP of the kth data bit u_k, $k = 1, 2, \ldots, K$, and y is the received codeword in noise, $y = c + n$. We assume u_k and the components of c take values in the set $\{\pm 1\}$ and n is a noise word. Knowledge of the APP-LLR is typically obtained using the MAP algorithm, which estimates Eq. (1) by incorporating the code's trellis as follows [1]

$$L(u_k \mid y) = \ln\left(\frac{\sum\limits_{(`s,s)\Rightarrow u_k=+1} \gamma_k(`s,s)\cdot\alpha_{k-1}(`s)\cdot\beta_k(s)}{\sum\limits_{(`s,s)\Rightarrow u_k=-1} \gamma_k(`s,s)\cdot\alpha_{k-1}(`s)\cdot\beta_k(s)} \right) \tag{2}$$

where $(`s,s)\Rightarrow u_k=+1$ is the set of transitions from the previous state $S_{k-1}=`s$ to the present state $S_k = s$ caused by $u_k = +1$, and similarly for $(`s,s)\Rightarrow u_k=-1$. In the log-MAP algorithm, the branch transition probability $\gamma_k(`s,s)$ is calculated in the logarithmic domain

$$\begin{aligned}\Gamma_k(s',s) &\equiv \ln(\gamma_k(`s,s)) \\ &= \ln(P(\mathbf{y}_k \mid \{`s,s\})\cdot P(u_k)) \\ &= \ln\left(C\cdot\exp(u_k L(u_k)/2)\cdot\exp\left(\frac{L_c}{2}\sum_{l=1}^{n} y_{kl}x_{kl}\right)\right)\end{aligned} \tag{3}$$

where $P(u_k) = \{\exp(-L(u_k)/2)/[1+\exp(-L(u_k))]\}\cdot\exp(u_k L(u_k)/2)$ is the prior probability of u_k and $L(u_k) = \ln(P(u_k=+1)/P(u_k=-1))$ is its LLR. x_{kl} and y_{kl} are the individual bits within the transmitted and received codewords for the kth information symbol, and $1/n$ is the code rate. The constant term C does not affect the summations in the numerator and denominator in Eq. (2) can be omitted. L_c is the channel reliability value defined as

$$L_c = 2a\frac{2E_b}{\sigma^2} \tag{4}$$

where E_b is the transmitted energy per bit, σ^2 is the noise variance and a is the fading amplitude. Similarly, the remaining two terms $\alpha_{k-1}(`s)$ and $\beta_k(s)$ in Eq. (2) can be calculated using a forward and backward recursion in the logarithmic domain as follows [5]

$$A_k(s) = \ln(\alpha_k(s)) = \ln\left(\sum_{`s}\alpha_{k-1}(`s)\cdot\gamma_k(`s,s)\right) \tag{5}$$

$$B_{k-1}(`s) = \ln(\beta_{k-1}(`s)) = \ln\left(\sum_{s}\beta_k(s)\cdot\gamma_k(`s,s)\right) \tag{6}$$

Thus the APP-LLR $L(u_k \mid y)$, which the log-MAP algorithm calculates is:

$$L(u_k \mid y) = \ln\left(\frac{\sum\limits_{\substack{(`s,s)\Rightarrow \\ u_k=+1}} \exp(\Gamma_k(`s,s)+A_{k-1}(`s)+B_k(s))}{\sum\limits_{\substack{(`s,s)\Rightarrow \\ u_k=-1}} \exp(\Gamma_k(`s,s)+A_{k-1}(`s)+B_k(s))} \right) \tag{7}$$

From the above, it is noticeable that the calculation of $\Gamma_k(s',s)$, $A_k(s)$, and $B_{k-1}(`s)$ requires knowledge of the channel reliability value L_c or SNR, and thus incorrect estimate of channel variance may result in significant performance degradation.

Max-Log-MAP decoder: To further reduce the complexity of the log-MAP algorithm, the max-log-MAP algorithm uses the following approximation

$$\ln\left(\sum_i \exp(x_i)\right) \approx \max_i(x_i) \quad (8)$$

Thus $A_k(s)$ and $B_{k-1}(`s)$ are calculated as follows:

$$A_k(s) \approx \max_{`s}\left(A_{k-1}(`s) + \Gamma_k(`s,s)\right) \quad (9)$$

$$B_{k-1}(`s) \approx \max\left(B_k(s) + \Gamma_k(`s,s)\right) \quad (10)$$

Finally, the max-log-MAP algorithm approximates the APP-LLR $L(u_k|y)$ as [5]

$$L(u_k \mid y) \approx \max_{\substack{(`s,s)\Rightarrow \\ u_k=+1}}\left(A_{k-1}(`s)+\Gamma_k(`s,s)+B_k(s)\right) - \max_{\substack{(`s,s)\Rightarrow \\ u_k=-1}}\left(A_{k-1}(`s)+\Gamma_k(`s,s)+B_k(s)\right) \quad (11)$$

It is observed that the correlation term $\sum_{l=1}^{n} y_{kl}x_{kl}$ in Eq. (3) is weighted by the channel reliability value L_c, but the latter will be factored out in Eq. (11) by the max(·) operation. Further, although the soft outputs $L(u_k \mid y)$ in Eq. (11) are scaled by L_c, taking the hard decisions will make the bit estimates independent of L_c. Therefore, we expect that serial turbo decoding using the max-log-MAP algorithm will be robust to errors in variance mismatch. Figure 1 illustrates the block diagram of the serial turbo codes under non-Gaussian noise interference.

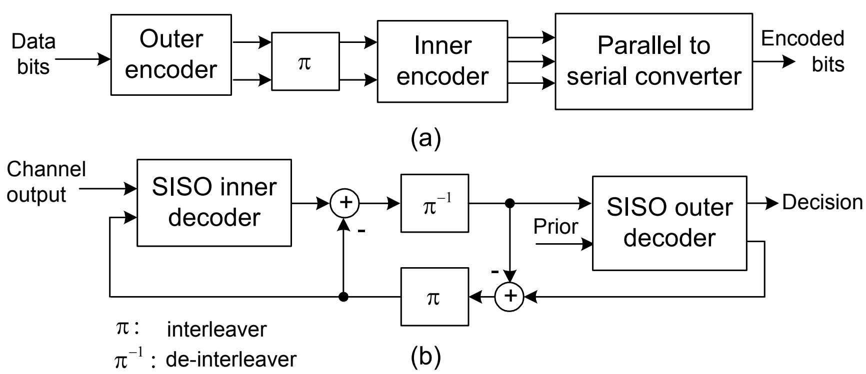

Fig. 1. Block Diagrams for the (a) Serial Turbo Encoder (b) Serial Turbo Decoder.

3. Impulsive Noise Model

Impulsive noise is a source of noise arising from hostile transmission medium such as PLC which appears in the channel in the form of short duration and ideally infinite amplitude pulse. Sharp impulse and random arrival of outliers as shown in Fig. 2 are induced to the communication systems and severely affects their stabilities and performances.

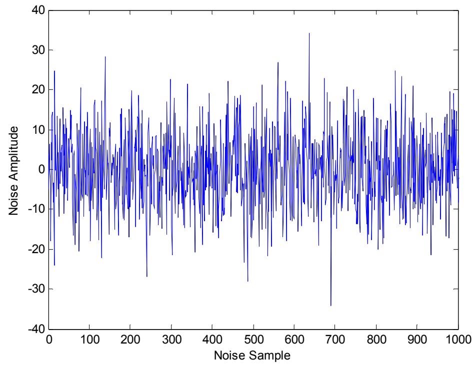

Fig. 2. An Example of Impulsive Noise Pattern (A = 0.1, Γ = 0.1).

The impact of impulsive noise is significant typical digital communication systems as it would cause unrecoverable corruption to the transmitted data. Impulsive noise has been studied intensively by various researchers to find a suitable mathematical model to describe the behaviour of such noise [10-12] and its impacts to the transmission medium [13]. Such an impulsive noise's mathematical model was used in modelling and simulation of SCCC in the light of finding a better and robust decoding strategy to enhance data integrity and reduce probability of errors.

Middleton's Class *A* noise model [10] is used to represent impulsive noise in statistical manner. And it can be categorised into class *A*, *B* and *C*. In our simulation, Class *A* noise model is adopted; The PDF (probability density function) of the impulsive noise *z* can be described as follows

$$p_A(z) = \sum_{m=0}^{\infty} \frac{e^{-A} A^m}{m!} \cdot \frac{1}{\sigma_m \sqrt{2\pi}} \exp\left(-\frac{z^2}{2\sigma_m^2} \right) \tag{12}$$

with

$$\sigma_m^2 = \sigma^2 \bullet \frac{(m/A) + \Gamma}{1 + \Gamma} \tag{13}$$

where A is the impulsive index, $\Gamma = \sigma_G^2 / \sigma_I^2$ is the GIR (Gauss-to-Impulse noise Power Ratio). Gaussian noise power is σ_G^2 and impulsive noise power is σ_I^2.

The degree of impulse received can be determined by impulsive index *A*. The impulsive noise tends to be more continuous if *A* is large and resembling to the AWGN noise. Otherwise, the smaller value of *A* will generate Class *A* impulsive noise. To ease the construction of Class *A* impulsive channel, as based on (12) statistical model which is distributed with Poisson distribution $\frac{e^{-A} A^m}{m!}$. The arrival of impulsive noise to the receiver can be characterised by the Gaussian PDF with variance as

$$\sigma_G^2 + \left(\frac{m\sigma_I^2}{A} \right) \tag{14}$$

where m is the number of impulsive noise sources and σ_G^2 is the background Gaussian noise.

4. Talwar Penalty Function

The performance of SCCC under impulsive noise is severely degraded as shown in the Fig. 3. Hence Talwar penalty was proposed in [9] to improve the estimation of channel bits and guards against outliers arising from the channel. The equation was shown in Eq. (15) to improve decoding performance with robust detection in SCCC

$$\rho_{talwar}(x) = \begin{cases} \frac{x^2}{2}, |x| \le v \\ \frac{v^2}{2}, |x| \ge v \end{cases} \tag{15}$$

where $v > 0$ is a clipping parameter against outlier. If the received symbols with amplitude less than v, then ρ_{talwar} leads to the LS estimator as shown in Fig. 3(a). For received symbols with amplitude larger than v, talwar function is used to clip and limit the input signal to guard against the entry of outlier as shown in Fig. 3(b). Therefore, it can be shown that the first condition is used to ensure good performance of SCCC in AWGN channel, and the second condition is used while the system functioning in the presence of impulsive noise.

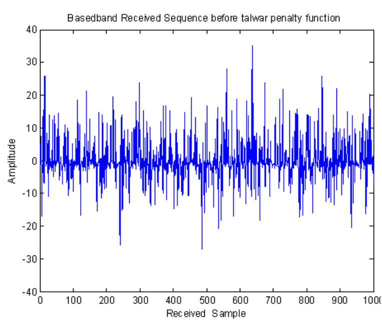

Fig. 3(a) Basedband Received Signal before Talwar Penalty Function.

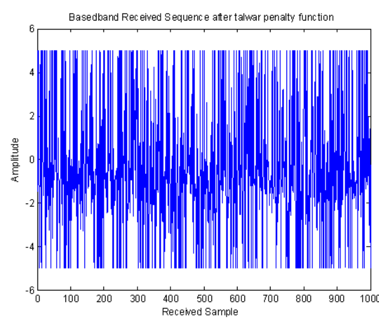

Fig. 3(b) Basedband Received Signal after Talwar Penalty Function, v = 5.

The use of talwar penalty function into SCCC can be given in the following conditional probability estimation as given in Eq. (16)

$$p(y_k|c_k)=\left(\frac{e^{-A}A^m}{m!}\right)^n\cdot\frac{1}{\left(\sigma_m\sqrt{2\pi}\right)^n}\exp\left(-\frac{L_c}{2}\sum_{m=0}^{\infty}\rho_{talwar}\left(y_k-c_k\right)\right) \tag{16}$$

5. Simulation and Results

The purpose of the simulation is to study the variance mismatch of SCCC in impulsive noise channel. Simulations were done in the conditions of original and proposed talwar enhanced SCCC in impulse noise channels. In order to examine the sensitivity of SCCC to variance mismatch, a series of simulations were carried out using the log-MAP and max-log-MAP component decoders. We form a rate 1/3 serial turbo code by cascading a rate ½ outer RSC code with a rate 2/3 RSC inner code. In addition, a frame length of K = 16384 information bits is used, and results were recorded after six decoding iterations. Note from Fig. 1(a) that the outer encoder feeds no bits directly to the channel, thus the outer decoder in Fig. 1(b) receives no samples directly from the channel. Thus knowledge variance estimate is only required for the inner decoding stage. The E_b/N_0 offset is defined as Eq. (17):

$$E_b/N_0 \text{ offset (dB)} = E_b/N_0 \text{ true (dB)} - E_b/N_0 \text{ estimated (dB)} \tag{17}$$

Figures 4 and 5 showed the BER performance of original SCCC for LOG-MAP and MAX-LOG-MAP algorithms in impulsive noise channel.

From Fig. 4, we can observe that the SCCC with LOG-MAP decoder that was initially designed for AWGN channel can not perform in impulsive noise channel and the decoding bits incurring high BER.

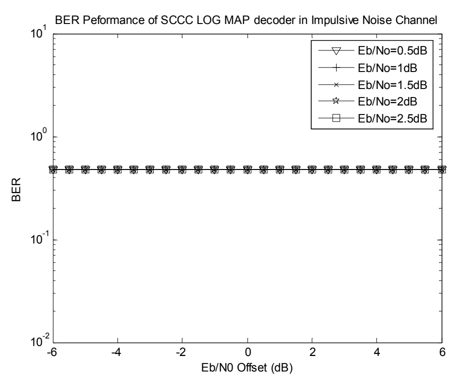

Fig. 4. Performance of SCCC without Talwar Penalty Function Using LOG-MAP Component Decoders in Impulsive Noise Channel (A = 0.1, Γ = 0.1).

Notice that as compared to results reported in [5-8] for PCCC and SCCC in AWGN channel, impulsive noise totally causes malfunction and handicaps the excellent decoding performance (Fig. 4) using the log-MAP component decoders. SCCC fails to decode impulse corrupted received sequence with respect to both underestimation and overestimation of channel variance which follows Poisson distribution. The performance of using MAX-LOG-MAP component decoders is shown in Fig. 5. MAX-LOG-MAP based SCCC thought is insensitive to variance mismatch in AWGN channel can merely perform slightly better than LOG-MAP component decoders in impulsive noise channel.

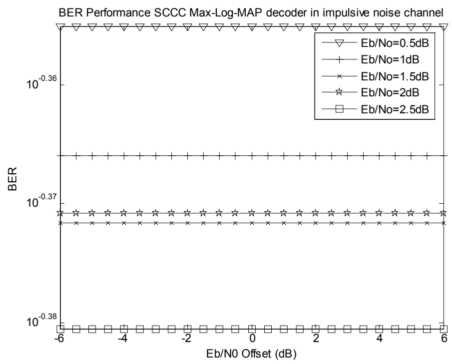

Fig. 5. Performance of SCCC without Talwar Penalty Function Using MAX-LOG-MAP Component Decoders in Impulsive Noise Channel (A = 0.1, Γ = 0.1).

Simulation of SCCC that was equipped with talwar penalty function is shown in Fig. 6. Talwar penalty function effectively guarded SCCC against outliers from impulsive channel. Hence, SCCC is performed in optimum condition as if it is ran in AWGN channels. Again from Fig. 6, we can clearly observe that despite the introduction of talwar penalty function, it doesn't modify the decoding capabilities of SCCC in impulse noise condition, and Shannon predicted excellent performance [1] can be perfectly obtained from such impulsive environment.

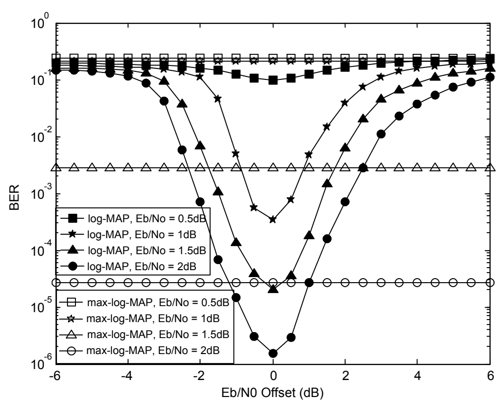

Fig. 6. Performance of Serial Turbo Codes with Talwar Penalty Function Using log-MAP and max-log-MAP Component Decoders in Impulsive Noise Channel (A = 0.1, Γ = 0.1).

6. Conclusions

The issue of variance mismatch on the performance of serial turbo codes (SCCC) using the log-MAP and max-log-MAP constituent decoders was investigated in impulsive noise channel. The log-MAP algorithm requires knowledge of the channel reliability for the metric calculation, whereas the max-log-MAP algorithm obviates this side information as a result of using an approximation. However, both traditionally designed algorithms failed to perform in impulse noise interference. Therefore, talwar penalty function was introduced and reported to effectively curb against outliers of impulse noise reported in [9]. By setting appropriate value for cutoff parameter v, the entries of undesired outliers from impulse noise is blocked and cut off prior to the computation of channel transition probability $\alpha(s', s)$. Therefore, LOG-MAP and MAX-LOG-MAP algorithms of SCCC can be used in determine extrinsic information L_e for inner and outer component decoders.

As for the increasing importance of using PLC for internet and information sharing [14], the study of variance mismatch for SCCC in impulsive noise channel for next generation communication system is crucial to the researchers and engineers in the field. Talwar penalty function enables and extends the excellent performance of MAP based SCCC to a new technology frontier.

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