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PREDICTION OF DYNAMIC RESPONSE OF STIFFENED RECTANGULAR PLATES USING HYBRID FORMULATION

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Abstract

A developed method, based on the varitional principles in combination with the finite difference technique, is applied to determine the dynamic characteristics of rectangular panels having stiffeners in both directions. The strain and kinetic energy for the plate and stiffener are expressed in terms of discrete displacement components using the finite difference method. Harmonic motion is assumed in order to eliminate the time dependency and exclude the transient response. The functional is minimized with respect to the discretised displacement components and the natural frequencies; and corresponding mode shapes are obtained from the solution of a linear eigenvalue problem. The natural frequencies and mode shapes of stiffened panels are determined for various boundary conditions. The results are included for a number of stiffened plates where the dimensions of the plate and stiffener cross section are chosen so that the mass of the plate-stiffener combination remains constant. The energy approach using the variational procedure yields acceptable results for the type of stiffened plates considered in this study. The results show that the natural frequencies can be increased by increasing the flexural rigidity of the stiffeners, and also the frequencies of two modes can be close or even be equal to the exact solution for a given size of stiffener.

Keywords: Hybrid formulation, Free vibration, Stiffened rectangular plates, Dynamic response.

1. Introduction

The dynamic behaviour of stiffened plates has been the subject of intensive study for many years. In order to reduce the vibration response of a rectangular plate, it is often necessary to change its natural frequencies by adding stiffeners. Stiffened panels are structural elements of practical importance in applications such as aircraft, ship, and bridge decks.

Nomenclatures

A	Cross section area of stiffener, mm^2
$[A]$	Coefficient matrix
a, b	Length and width of a plate respectively, mm
D	Flexural rigidity for an isotropic plate, mm^3
d	Width of a stiffener, mm
E	Elastic module, N/mm^2
e	Stiffener eccentricity, mm
F	Quadratic function in discrete displacement variables, or depth of a stiffener, mm
G	Modulus of elasticity in shear, , N/mm^2
H	Linear function of the square of discrete velocities
h	Thickness of the plate, mm
I_p	Polar mass moment of inertia , mm^4
I_s	Second moment of inertia, mm^4
J	Torsion constant
k_r	Radius of gyration, mm
MM, NN	Number of rows and columns in the node set respectively
m, n	Mode numbers
N	Number of equations
p,q,r,s	Unequal mesh interval parameters in x and y directions
Q	Non-dimensional parameter , a/h
T	Time, s
T_p, T_S	Kinetic energy of plate and stiffener respectively
U_P, U_S	Strain energy of plate and stiffener respectively
u, v, w	Displacement components
W	Displacement component independent of time
w^1	Non-dimensional displacement component
x, y, z	Rectangular coordinates

Greek Symbols

α	Ratio of finite difference increments in orthogonal directions
γ	Aspect ratio, b/a
Γ	Warping constant
ζ, η	Non-dimensional coordinate, x/a and y/a
$\Delta\zeta$, $\Delta\eta$	Finite difference increments
θ	Angular displacement
λ	Eigenvalue
ρ	Mass density, kg/mm^3
υ	Poisson's ratio.
ϕ	Non-dimensional parameter , d/a
ψ	Non dimensional parameter , f/h
ω	Frequency of vibration, rad/s
$\wp$	Lagrangian

The existing analytical procedures appear to be confined to various kinds of approximations of the system. An obvious and commonly used method is to assume that the structure is adequately approximated by an orthotropic plate. This idealization assures that the beams which occur at discrete intervals can be replaced by a continuum. Gorman [1] has evaluated the free vibration of cantilever plates with rectangular orthotropy. Initially, a theoretically "exact" solution to the beam-plate structure was found by combining the classical isotropic plate theory for the plate and beam theory for the beams. This solution was compared with the results of the orthotropic plate theory.

However, for a particular class of stiffened plates, a more rigorous analysis is presented by Molaghasemi [2]. The problem of free vibrations of a rectangular, composite base plate or panel stiffened by a bonded, eccentral stiffening plate strip was considered by Yuceoglu [3]. Long [4] investigated rectangular plates with stiffening in one direction and assumed that the in-plane displacements in the direction normal to the direction of stiffening were negligible. Free vibrations of ribbed plates by a combined analytical numerical method were investigated by Lorenzo and Massimo [5]. Nagino et al [6] determined the three-dimensional free vibration analysis of isotropic rectangular plates using the B-spline Ritz method. Kirk [7] determined the natural frequencies of the first symmetric model of a simply supported rectangular stiffened plate. The ratio of a frequency-stiffened plate to a frequency-unstiffened plate of equal mass was determined by the Ritz method using a two-term solution for a wide range of ratios of plate dimensions and stiffener depth/plate thickness ratios.

A finite element model was proposed by Holopainen [8] for free vibration analysis of eccentrically stiffened plates. In his study, the formulation allows the placement of any number of arbitrarily oriented stiffeners within a plate element without disturbing their individual properties. A plate-bending element consistent with the Reissner-Mindlin thick plate theory was employed to model the behaviour of the plating. A stiffener element, consistent with the plate element, was introduced to model the contributions of the stiffeners. The applied plate-bending and stiffener elements were based on mixed interpolation of tensorial components, to avoid spurious shear locking and to guarantee good convergence behaviour. The typical stiffened plate considered here is composed of two stiffeners which are located eccentrically on the axes of symmetry of the plate. Although stiffeners with rectangular cross section have been discussed here, they may have any cross sectional configuration of known force-deflection properties.

The energy approach by employing varitional procedure in conjunction with the finite difference method was applied to the plates with cutouts by Aksu and Ali [9]. This approach has also been developed for the calculation of natural frequencies and mode shapes of stiffened plates.

The strain and kinetic energy for a plate and stiffener are expressed in terms of discrete displacement components and the natural frequencies and mode shapes of the stiffened plate are then obtained as the solutions of an algebraic eigenvalue problem. The derivatives appearing in the functional are replaced by finite difference equations with unequal intervals. The effect of finite difference formulation with unequal intervals was studied by Aksu and Ali [10]. The numerical approach presented here obviates the necessity of simply supported edge condition.

As opposed to the plates with stiffeners in one direction only, here both the in-plane displacements u and v have to be considered. If all the displacements u, v, and w are included in the analysis, then the natural frequencies and mode shapes would be obtained as a large linear algebraic eigenvalue system. This may prove to be an uneconomic method. However, it is possible to obtain approximately natural frequencies and mode shapes by neglecting in-plane displacements. The ratio of the frequencies of the stiffened and unstiffened plates of equal mass has been determined for a class of plates considering the mass as an important factor for designers.

2. Theory

A stiffened plate, typical of the class considered here, is shown in Fig.1. It is assumed that stiffeners are an integral part of the stiffened plate, i.e., no slippage occurs between the plate and the stiffener.

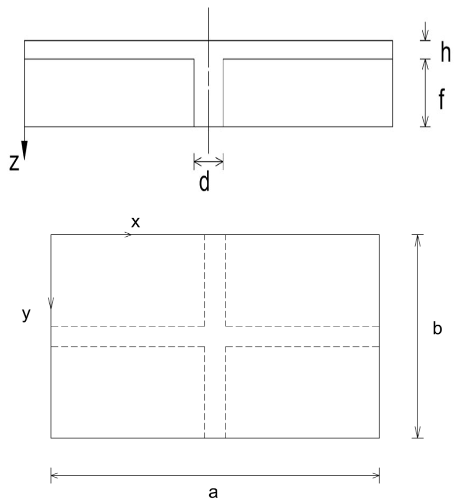


Fig. 1. Typical Stiffened Plate.

The strain energy of an isotropic plate due to bending is given by Timoshenko and Krieger [11]:

$$U_b = \frac{1}{2} D \int_0^b \int_0^a \left[\left(\frac{\partial^2 w}{\partial x^2} \right)^2 + 2\upsilon \left(\frac{\partial^2 w}{\partial x^2} \right) \left(\frac{\partial^2 w}{\partial y^2} \right) + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + 2(1-\upsilon) \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] dxdy \tag{1}$$

where w is the displacement component independent of time. Since the in-plane deformations are neglected in order to reduce the number of unknown displacements per node, some of the boundary conditions such as for simply supported edges, in-plane motion of the plate as it deflects, can not be satisfied. If the plate boundaries are restrained against in-plane motion, the stiffener bending is assumed to be about an axis in the mid-plane of the plate which implies an upper bound to the stiffener flexural rigidity. Let A be the cross sectional area of the stiffener, I its moment of inertia about its centroidal axis and e the stiffener eccentricity, i.e., the distance from the stiffener centroidal axis to the mid-plane of the plate. The strain energy due to bending of the beam is expressed as

$$U_b = \frac{1}{2} EI_s \int_0^b \left(\frac{\partial^2 w}{\partial y^2} \right)^2 \tag{2}$$

$$I_s = I + e^2 A$$

where I_s is the second moment of inertia about an axis in the mid-plane of the plate. In addition, the stiffener undergoes non-linear twisting about an axis, $x = a/2$, lying in the middle surface of the plate. The torsion of the stiffener leads to warping of the cross-sections along the stiffener length. Thus in addition to the strain energy due to torsion, that of the twisted stiffener is given by

$$U_t = \frac{1}{2} GJ \int_0^b \left(\frac{\partial \theta}{\partial y} \right)^2 dy + \frac{1}{2} E\Gamma \int_0^b \left(\frac{\partial^2 \theta}{\partial y^2} \right)^2 dy \tag{3}$$

where J is the torsional stiffness constant and Γ is the warping constant. The rotation θ can be replaced by $\left(\frac{\partial w}{\partial x} \right)$. The kinetic energies of the plate and the stiffeners are respectively

$$T_P = \frac{1}{2} \rho h \int_0^b \int_0^a \left(\frac{\partial w}{\partial t} \right)^2 dxdy \tag{4}$$

$$T_s = \frac{1}{2} \int_0^b \left[\rho A \left(\frac{\partial w}{\partial t} \right)^2 + I_P \left(\frac{\partial^2 w}{\partial x \partial t} \right)^2 \right] dy \tag{5}$$

where I_P is the polar mass moment of inertia. The strain and kinetic energies based on the following non-dimensional parameters, can be expressed as

$$z = \frac{x}{a}, \eta = \frac{y}{a}, w^1 = \frac{w}{a}, \gamma = \frac{b}{a}$$

$$U_p = \frac{1}{2} D \int_0^\gamma \int_0^1 \left[\left(\frac{\partial w^1}{\partial \zeta^2} \right)^2 + 2\nu \left(\frac{\partial w^1}{\partial \zeta^2} \right) \left(\frac{\partial w^1}{\partial \eta^2} \right) + \left(\frac{\partial w^1}{\partial \eta^2} \right) + 2(1-\nu) \left(\frac{\partial^2 w^1}{\partial \zeta \partial \eta} \right)^2 \right] d\zeta d\eta \tag{6}$$

The strain energy of the stiffener centrally located such as in η direction,

$$U_s = \frac{1}{2} \int_0^\gamma \left[\frac{EI_s}{a} \left(\frac{\partial^2 w^1}{\partial \eta^2} \right)^3 + \frac{GJ}{a} \left(\frac{\partial^2 w^1}{\partial \zeta \partial \eta} \right)^2 + \frac{E\Gamma}{a^3} \left(\frac{\partial^3 w^1}{\partial \zeta \partial \eta^2} \right)^2 \right] d\eta \tag{7}$$

$$T_P = \frac{1}{2} a^4 h \rho \int_0^\gamma \int_0^1 \left(\frac{\partial w^1}{\partial t} \right)^2 d\zeta d\eta \tag{8}$$

$$T_s = \frac{1}{2} \int_0^\gamma \left[a^3 \rho A \left(\frac{\partial w^1}{\partial t} \right)^2 + I_P a \left(\frac{\partial^2 w^1}{\partial \zeta \partial t} \right)^2 \right] d\eta \tag{9}$$

For a rectangular section integral stiffener situated on one face of the plate, the second moment of area about the middle surface of the plate is:

$$I_s = \frac{dh^3}{3}\left(\psi^3 + 1.5\psi^2 + 0.75\psi\right) \tag{10}$$

where $\psi = f/h$. Defining $\phi = d/a$ then

$$\frac{EI_s}{a} = \frac{1}{3}E\phi h^3\left(\psi^3 + 1.5\psi^2 + 0.75\psi\right) \tag{11}$$

In the case of a rectangular section stiffener of width d and depth f, it is assumed that for the stiffener to be effective, its depth must be equal to, or greater than, its width. The Saint- Venant torsion constant J is taken from Dowty [12] and is as follows,

$$\frac{GJ}{a} = \frac{G\psi(a\phi)^3}{16}\left\{5.333 - \left(3.36\frac{Q\phi}{\psi}\right)\left[1 - 0.0833\frac{Q\phi^4}{\psi}\right]\right\} \tag{12}$$

where $Q = a/h$ and the warping constant Γ is given by:

$$\frac{E\Gamma}{a} = \frac{Ea\phi\psi^3}{36Q^3}\left[\left(1 + \frac{0.5}{\psi}\right)^3 - \left(\frac{0.5}{\psi}\right)^3\right] \tag{13}$$

The polar mass moment of inertia I_p can be expressed as

$$I_P = \rho A k_r^2$$

$$aI_P = \frac{\rho a^4 h\phi\psi}{4Q^2}\left[\frac{1}{3}\left(Q^2\phi^2 + 4\psi^2\right) + 2\psi + 1\right] \tag{14}$$

where k_r is the radius of gyration of the stiffener cross- section about the axis of rotation.

The integrals in the strain energy expressions are replaced by finite approximating sums based on the mesh covering the plate. Subsequently, with the use of standard finite difference formulas, the total strain energy of the plate and beam can be expressed as a quadratic form F in terms of the discrete displacement variables.

$$F(w^1{}_{i,j}) = U_P + U_S \tag{15}$$

Similar approximations reduce the kinetic energy to a linear function H of the squares of discrete velocities,

$$H\left[\left(\frac{\partial w^1}{\partial t}\right)_{i,j}^2\right] = T_P + T_s \tag{16}$$

Applying Euler's necessary condition to minimize the total energy of the system gives the relations

$$\frac{d}{dt}\left(\frac{\partial \wp}{\partial \dot{w}^1}\right)-\left(\frac{\partial \wp}{\partial w^1}\right)=0 \tag{17}$$

where Lagrangian is

$$\wp = U_P + U_S - T_P - T_S \tag{18}$$

The solution of harmonic motion may be assumed in the form of the product:

$$w^1(\zeta,\eta,t)=(A\cos\omega t+B\sin\omega t)W(\zeta,\eta) \tag{19}$$

Equation (19) can be expressed as a matrix eigenvalue problem as follows

$$[A]\{X\}=\lambda[B]\{X\} \tag{20}$$

$$\lambda = a^4 \rho h \omega^2$$

A computer program was developed for this analysis. The program carried out the substitution of finite difference equations and performed the partial differentiation required by Eq. (19).

3. Finite Difference Formulation

A finite difference formulation of the energy integral expression due to bending of the plate was obtained by second order Lagrangian interpolation polynomial by Aksu and Ali [8].

Lagrange interpolation polynomial was also used to determine the finite difference equations for the strain energy expression of the stiffener Eq. (7). The strain energy associated with bending and warping was computed as the sum of the strain energies summed over the set of non overlapping nodal sub domains as shown in Fig. 2.

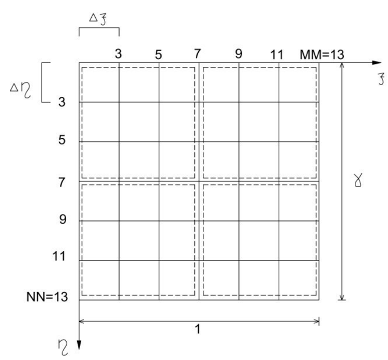


Fig. 2. Node Set of ss-ss-ss-ss Stiffened Plate.

The finite difference equations for an interior node are given by:

$$\left(\frac{\partial^2 w}{\partial \eta^2}\right)_{i,j}=\frac{2}{(\alpha\Delta\zeta)^2}\left[\frac{W_{i-1,j}}{p_i(p_i+s_i)}-\frac{W_{i,j}}{p_i s_i}+\frac{W_{i+1,j}}{s_i(p_i+s_i)}\right] \tag{21}$$

$$\left(\frac{\partial^3 w}{\partial\zeta\partial\eta^2}\right)_{i,j} = \frac{2}{\alpha^2\Delta\zeta^3}\left\{\begin{array}{l} \frac{1}{p_i(s_i+p_i)}\left[-\frac{r_j}{q_j(q_j+r_j)}W_{i-j,j-1}+\frac{r_j-q_j}{r_jq_j}W_{i-1,j}+\frac{q_j}{r_j(q_j+r_j)}W_{i-1,j+1}\right]-\frac{1}{p_is_i} \\ \left[-\frac{r_j}{q_j(q_j+r_j)}W_{i,j-1}+\frac{r_j-q_j}{r_jq_j}W_{i,j}+\frac{q_j}{r_j(q_j+r_j)}W_{i,j+1}\right]+\frac{1}{s_i(p_i+s_i)} \\ \left[-\frac{r_j}{q_j(q_j+r_j)}W_{i+1,j-1}+\frac{r_j-q_j}{r_jq_j}W_{i+1,j}+\frac{q_j}{r_j(q_i+r_j)}W_{i+1,j+1}\right] \end{array}\right\} \quad (22)$$

where the intervals $\Delta\zeta$ and $\Delta\eta$ are defined when the plate is divided into (MM-1) and (NN-1) and subdivisions in each coordinate direction respectively:

$$\Delta\zeta = \frac{1}{MM-1}, \Delta\eta = \frac{1}{NN-1}, \alpha = \frac{\gamma(MM-1)}{NN-1} \quad (23)$$

For the strain energy due to torsion, the nodal subdomains for an interior node is shown in Fig. 3 It is possible to use two sets of interlacing grids in order to reduce the discretization error.

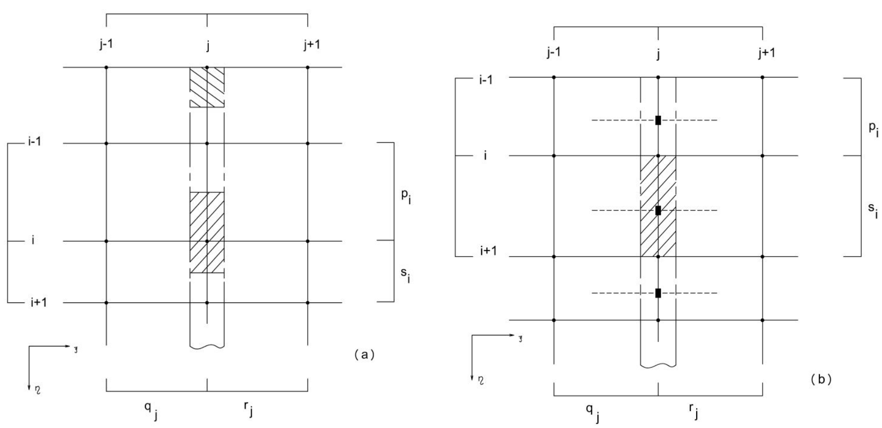


Fig. 3. Nodal Subdomains for Stiffener.

The finite difference formulations of an interior node $\left(i+\frac{1}{2}, j\right)$ with displacement variable $W(\zeta, \eta)$ can be expressed as:

$$\left(\frac{\partial^2 W}{\partial\zeta\partial\eta}\right)_{i+\frac{1}{2},j} = \frac{1}{s_i\alpha\Delta\zeta^2}\left[\begin{array}{l} \frac{r_j}{q_i(q_j+r_j)}W_{i,j-1}-\frac{r_j-q_j}{r_jq_j}W_{i,j}-\frac{q_j}{r_j(q_j+r_j)}W_{i,j+1}-\frac{r_j}{q_j(q_j+r_j)}W_{i+1,j-1} \\ +\frac{r_j-q_i}{r_jq_j}W_{i+1,j}+\frac{q_j}{r_j(q_j+r_j)}W_{i+1,j+1} \end{array}\right] \quad (24)$$

The algebraic eigenvalue problem defined in Eq. (24) has certain disadvantages. It may be considered that the solution of the eigenvalue problem is uneconomical due to computation time and core memory requirements. However, in certain modes, it is possible to transform a square matrix [B] into a diagonal matrix. The square matrix [B] is obtained due to the effect of the torsional kinetic energy of the stiffener. Since in the determination of the first symmetric mode, the stiffener is subjected to bending only, the torsional kinetic energy need not be included in the analysis. Then the matrix [*B*] is reduced automatically to a diagonal matrix. However, in certain modes, such as in the first antisymmetric mode, the stiffener is subjected to twisting, so the procedure described in Aksu [8] [*B*]is applied in the formation of the diagonal matrix [*B*].

4. Results and Discussions

Kirk [7] has determined the natural frequencies of the first symmetric and first anti-symmetric modes of a simply supported rectangular plate reinforced by a single stiffener placed along one of its centre lines. The ratio of the frequency of stiffened plate to the unstiffened plate of equal mass was determined by Ritz method using a two term solution in which the in plane displacements were completely neglected. Since the cross-stiffened plates are also studied by neglecting in plane motion, this method was tested by comparing the Ritz solution for a stiffened plate with a single stiffener.

Fundamental frequency for a stiffened square plate; a = 635 mm, Q = 250, ϕ = 0.01 and ψ = 5 was determined as f = 55.78 Hz by the method proposed in this paper. The thickness C_o for an unstiffened plate was obtained by equating the masses of the stiffened and unstiffend plate as

$$\rho a\left(ah + A_s\right) = \rho a^2 h_0 \tag{25}$$

$$h\left(1+\phi\psi\right) = h_0 \tag{26}$$

The frequencies of the modes of vibration of an unstiffened square isotropic plate of uniform thickness h_o = 1.05h are given by Kirk [7]

$$f_0 = \frac{h_0}{2a^2}\sqrt{\frac{E}{12\left(1-\upsilon^2\right)}}(m^2+n^2) \tag{27}$$

for the fundamental mode (m = 1, n = 1), the frequency was found to be f_o = 32.32 Hz. Hence, the ratio of the frequency of stiffened plate to the unstiffened plate is f/f_o = 1.728. By the Ritz method, this ratio was found to be 1.74 in Kirk [7]. Similarly for the first antisymmetric mode f/f_o was determined as 0.97 and the Ritz solution respectively.

In Table 1, the natural frequencies of a cross stiffened plate as shown in Fig. 2 are given for various values of ψ when all edges are simply supported for a cross stiffened square plate a = 635 mm, Q = 250, ϕ = 0.01 and different thickness h_o. Presented results in the Table 1 have quit well agreement with the results of the exact solution when two modes are chosen, and thereby confirm the validity of the proposed procedure. Additionally, the natural frequencies of unstiffened plates with equal mass have been included.

The curves presented in Fig. 4 show the variation of frequencies with ψ. Fundamental frequency increases sharply in the interval $\psi = 0-10$, then gradually approaches to a maximum as $\psi \to \infty$ which corresponds to the stiffener becoming a clamped edge. So, the rate of increase in natural frequencies decreases after $\psi = 10$, and the natural frequency of the second mode is slightly higher than the fourth mode in the interval $\psi = 5$ and $\psi = 10$ approximately.

Hence, each quarter of the plate in this interval can be considered as an independent plate section having a fundamental mode which is lower than the natural frequency of the second mode of the structure.

Table1. Frequencies (Hz) for a Cross Stiffened Square Plate a = 635 mm, Q = 250, ϕ = 0.01 with Various Values of ψ and Different Thickness h_o and all Edges are simply Supported.

Mode	ψ	Cross Stiffened Plate		Square Plate	
		N = 49 NN = 9, MM = 9	N = 121 NN = 13, MM = 13	h_o	Exact
m = 1, n = 1	5	70.94	71.83	1.1h	33.86
m = 2, n = 1		131.23	136.79		84.67
m = 2, n = 2		130.38	135.53		135.47
m = 1, n = 1	10	127.73	133.64	1.2h	36.94
m = 2, n = 1		150.33	153.63		92.36
m = 2, n = 2		149.21	153.21		147.78
m = 1, n = 1	15	142.24	151.67	1.3h	40.02
m = 2, n = 1		160.09	164.83		100.06
m = 2, n = 2		168.36	167.39		160.10
m = 1, n = 1	20	145.34	155.50	1.4h	43.10
m = 2, n = 1		167.33	169.81		107.76
m = 2, n = 2		183.77	178.03		172.40

Referring to Fig.5, the frequency ratios of a stiffened plate to an unstiffened plate of equal mass for the range of values ψ, the curves are seen to rise to their maximum and then fall. For $m = n = 2$ roughly in the interval $\psi = 0 - 10$ the rate of increase in f/f_o is less than $m = n = 1$ and $m = 2$, $n = 1$. Transforming some of the unstiffened plate material into stiffeners, which is in effect equivalent to removing a layer of material of uniform thickness from the plate and forming it into stiffeners, will concentrate mass where it is most effective in lowering the frequency ratio. Reducing the plate thickness will lower the frequency ratio as well.

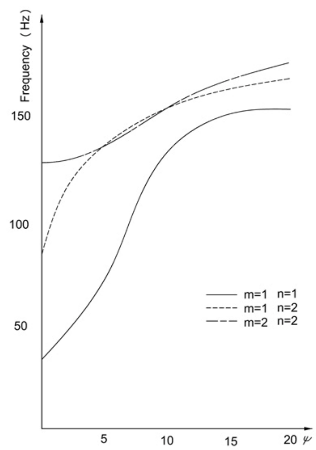


Fig. 4. Natural Frequencies with Various Values of ψ.

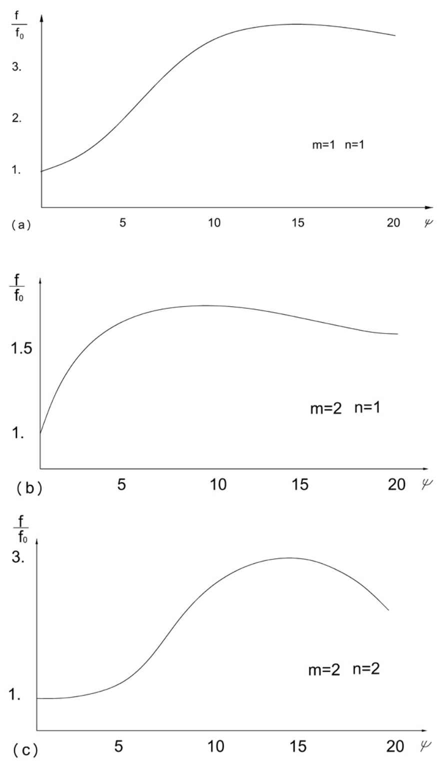
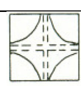
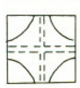
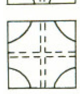
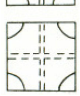


Fig. 5. Frequency Ratios of Stiffened Plate to Unstiffened Plate of Equal Mass versus ψ.

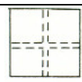
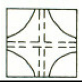
In Table 2, the natural frequencies of the fifth mode are given for various values of ψ. By increasing the flexural rigidity ψ of the stiffener, the nodal lines move gradually towards the corner, and also the natural frequencies are increased.

Table 2. Fifth Natural Frequency (Hz) with Various Values of ψ.

Nodal Patterns	ψ	Finite Differences	
		N = 49 NN = 9, MM = 9	N = 121 NN = 13, MM = 13
	**5**	175.87	186.78
	**10**	209.55	217.72
	**15**	260.33	280.85
	**20**	276.97	312.96

In Table 3, the natural frequencies and nodal pattern are given for a cross stiffened square plate a = 635 mm, Q = 250, ϕ = 0.01 when all edges are clamped. More details have been shown in Fig. 2.

Table 3. Frequency (Hz) for a Cross Stiffened Plate a = 635 mm, Q = 250, ϕ = 0.01 with all Edges Clamped.

Nodal Patterns	ψ	Finite Differences	
		N = 49 NN = 9, MM = 9	N = 121 NN = 13, MM = 13
	**5**	143.38	152.84
	**10**	166.43	182.57
	**15**	163.74	176.79
	**20**	222.60	243.45

5. Conclusion

An energy-based method has been developed for analysis of the dynamic characteristics of rectangular panels having stiffeners in both directions. The method is based on the variational principles in conjunction with finite difference method. The energy approach using the variational procedure yields acceptable results for the type of stiffened plates considered in this study. The natural frequencies can be increased by increasing the flexural rigidity of the stiffeners. The frequencies of two modes can be close or even be equal to the exact solution for a given size of stiffener. The method proposed here is approximate; however, the results can be used as pointers for practical applications. The accuracy can be improved by including the displacements u and v (see the notations) into the analysis.

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